International Trade, Volatility, and Income Differences

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Abstract

This paper presents a unified explanation for two puzzles in trade and development: developing countries' unexplained high export costs and the negative correlation between firm-level volatility and exports. I propose a novel general equilibrium model featuring heterogeneous firms, variable demand elasticity, and exporter dynamics. Under this framework, downturn profit losses outweigh boom gains, reversing the "Oi-Hartman-Abel" effect typically found in heterogeneous firm models. Consequently, firm-level volatility discourages exporting and investment, significantly reducing income and aggregate exports. Quantitatively, the model explains a substantial share of the puzzling cross-country export cost differences and the observed negative relationship between firm-level sales volatility with both income and exports. *JEL* Codes: F10, F12, F14, F23, F43, F63, O11, O19, O24, L11

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1 Introduction

The stark disparity in living standards across the globe is mirrored in international trade patterns. Developed nations engage in more trade, with a greater number of larger and more persistent exporters (Besedeš, 2011; Fernandes et al., 2015; Hummels et al., 2005). Existing research explains these patterns by estimating higher costs to export in developing economies (Blum et al., 2019; de Sousa et al., 2012; Waugh, 2010). However, the underlying causes of these higher costs remain unclear, preventing the development of effective policies to enable developing economies to capture the full benefits of global market access. I tackle this question by showing how the higher firm-level productivity volatility, typically more prevalent in developing economies, serves as a significant obstacle to both economic development and international trade.

My theory bridges two standing puzzles in the literature. First, it explains the observed negative correlation between firm-level sales volatility and total trade, which standard firm-heterogeneous models with sunk costs investment struggle to reconcile (Alessandria et al., 2015; Baley et al., 2020). Second, it offers a novel explanation for the persistently high export costs faced by developing economies, which previous literature has identified but not fully explained (Blum et al., 2019; de Sousa et al., 2012; Fernandes et al., 2015; Fieler, 2011; Waugh, 2010). I argue that the higher prevalence of firm-level volatility in developing economies reduces exports and income and thus explains their lower trade engagement. Because standard models omit the negative impact of firm-level volatility on total trade, they inadvertently overestimate export costs for these economies to reconcile the model with observed trade patterns.

The dampening effect of volatility on exports and overall income is rooted in two crucial factors, typically absent in standard models with firm heterogeneity but present in the micro-level data: variable price elasticity of demand (henceforth, price elasticity) and dynamic export decisions. When the price elasticity increases sufficiently with firm prices, reductions in profits during downturns outweigh potential gains in upswings. This generates a negative impact of firm productivity volatility on expected profits and total sales, reversing the standard "Oi-Hartman-Abel" effect, where higher volatility typically increases expected returns and production.[1] Dynamic export decisions further amplify this negative relationship: domestic firm-level

[1] The "Oi-Hartman-Abel" refers to the case in which higher volatility increases the firms' expected return because the expected profits in good times compensate for the profits of bad times (Bloom, 2013).

volatility discourages firms from investing in expanding foreign sales, hindering their growth and resilience, ultimately resulting in lower exports and income. These combined channels have sizable quantitative implications for income differences and trade patterns across countries.

I start with a simplified yet generic version of the model and derive four key theoretical results. First, I demonstrate that a mean-preserving spread in firms' productivity reduces total trade if firms' revenue functions are concave in productivity. Second, I show that this concavity critically depends on how demand elasticity varies with firm prices: sufficiently variable price elasticity can generate concave revenue functions. Third, when exporters invest in customer capital to increase foreign sales, this negative impact of firm-level volatility on total trade is intensified due to reduced exporter growth. Fourth, a misspecified revenue function leads the the model to estimate increases in export costs as firm productivity volatility rises, to match the observed data. These last two results provide clear testable implications that I confront with data before performing quantitative analyses.

I test the model's main assumptions and predictions using micro-level data from Colombia. My empirical findings are consistent with firms' facing variable price elasticity and exporter dynamics driven by shifts in foreign demand, similar to the findings in Fitzgerald. et al. (2024) and Steinberg (2023) for other countries. These two findings constitute the key microeconomic channels of the mechanism. I also document that, as predicted by the model, exporters facing higher firm-level sales volatility experience slower growth.

Subsequently, using cross-country data, I document several conditional correlations that inform the soundness of the model's quantitative implications. First, I document a negative relationship between the volatility of the idiosyncratic component of firm-level shocks and GDP per capita.[2] Then, I show that export costs, identified using standard gravity estimation techniques (Eaton et al., 2002; Fally, 2015; Waugh, 2010), are positively correlated with firm-level sales volatility, even after controlling for economic development. Finally, I find that conditioning on firm-level volatility reduces the estimated negative relationship between development and export costs by approximately 30%. These correlations do not imply causality, but they provide crucial

[2]My measures of firm-level sales volatility are based on firms' idiosyncratic shocks, abstracting from industry or aggregate drivers.

moments for testing the model's ability to explain relationships that have puzzled the literature (Alessandria et al., 2015; Blum et al., 2019; de Sousa et al., 2012; Fieler, 2011; Waugh, 2010).

I use a general equilibrium model to assess the quantitative relevance of the proposed mechanism in explaining the documented cross-country relationships, all of which are determined in equilibrium. In the full quantitative model, firms face productivity shocks, self-select into exporting (as in Melitz, 2003), and incur a sunk investment to build customer capital abroad similar (as in Fitzgerald. et al., 2024; Steinberg, 2023). However, unlike these previous works, I introduce variable price elasticity, akin to Klenow et al. (2016) and Edmond et al. (2023). The quantitative model nests four different models, whose combinations are defined by: (1) the existence or absence of exporter dynamics and (2) the existence or absence of variable price elasticities. To discipline the parameters underlying these microeconomic behaviors, I match the micro-level estimates in the model through indirect inference.

The quantitative results provide strong support for the proposed mechanism. Specifically, my proposed model predicts negative correlations between firm-level sales volatility, GDP per capita, and total trade, as well as an income-exports relationship consistent with the data. Notably, a reduction in firms' productivity volatility that moves a country from the median to the first quartile of firm-level sales volatility increases exports by 84% However, when variable markups are shut down, the model model predicts a positive correlation between firm-level sales volatility, total trade, and income, contrary to empirical evidence. This demonstrates that sunk cost investment in customer capital alone is insufficient to explain the observed negative volatility-export relationship. Nevertheless, when variable markups are included, sunk investment in customer capital becomes important. The investment decision amplifies the negative effects of volatility changes on total trade by almost 60%.

The proposed mechanism also has significant implications for the effect of firm productivity volatility on income. Consider, for example, an increase in firm productivity volatility moving a country from the first or second quartile to the third quartile of the firm-level sales volatility; this would reduce GDP per capita by about 35% and 25%, respectively. This highlights the substantial role of the proposed mechanism in understanding how differences in firm-level volatility affect economic development.

In essence, this paper identifies a novel mechanism through which firm productivity volatility and uncertainty discourage firms from investing in expanding over their life cycle, reducing both income and total trade. Firm-productivity volatility creates significant trade barriers that negatively correlate with income and are distinct from traditional, trade policy-driven barriers. The findings yield two important policy implications. First, they challenge the traditional view that gains from international trade solely depend on trade-policy changes, demonstrating that non-trade policies reducing firm-level volatility can significantly foster trade and development. Second, they highlight the challenges developing nations may face in reducing these non-trade policy barriers driven by firm-level volatility; history suggests that reducing them has been particularly challenging.

Literature. The paper relates to several strands of literature at the intersection of macroeconomics, international trade, firm dynamics, and development.

This paper is the first to show that cross-country differences in firm-level productivity volatility explain a significant proportion of the puzzlingly high export costs faced by developing economies. Existing literature extensively documents this positive correlation between estimated export costs and lower levels of development (e.g. de Sousa et al., 2012; Waugh, 2010).By construction, these estimated costs are not driven by productivity differences; however, their underlying drivers, which must negatively correlate with development, remain largely unexplained. While several mechanisms have been proposed, they only partially account for these differences (e.g. Blum et al., 2019; Fieler, 2011; Waugh, 2010). I quantitatively show that the cross-country differences in the volatility of firm productivity generate an export cost-income relationship consistent with observed data patterns.

Second, I highlight the significant impact of firm-level idiosyncratic productivity volatility on GDP per capita. While existing literature largely focuses on macro and sectoral volatility (e.g. Aghion et al., 2010; Imbs, 2007; Koren et al., 2007; Ramey et al., 1995), I identify micro-level volatility —abstracting from broader fluctuations—as an important distinct driver for development. [3]

Third, I contribute to the literature on exporter and firm dynamics (Eaton et al., 2007; Fitzgerald. et al., 2024; Ruhl et al., 2017; Steinberg, 2023). I document the negative impact of domestic

[3] I abstract from financial frictions in my model, as my empirical evidence, consistent with Leibovici (2021), suggests they are not a primary driver of export cost differences in development, despite being important for development (Aghion et al., 2010)

firm-level volatility on the growth of new exporters and a novel theoretical framework that aligns with these empirical findings. Allowing for exporter investment in customer capital as in Fitzgerald. et al. (2024), together with variable price elasticity of demand (as in Arkolakis et al., 2017; Edmond et al., 2023), is crucial to replicate the data. I extend the indirect utility function based on Kimball (1995), Klenow et al. (2016), Arkolakis et al. (2017), and Edmond et al. (2023), in a way that allows for tractable endogenous demand shifters together with heterogeneous markup responses.

Fourth, I contribute to the literature on firm-level uncertainty and trade, as well as the literature on investment under uncertainty. Traditionally, trade models struggle to explain the observed negative correlation between firm-level volatility and total trade (e.g. Alessandria et al., 2015; Baley et al., 2020; Handley et al., 2022), often relying on risk aversion to address this puzzle (e.g. Esposito, 2022; Handley et al., 2022; Limão et al., 2015). I present a novel micro-founded mechanism that resolves this puzzle without such assumptions. This mechanism, centered on firm-level variable price elasticity, has broad applicability in understanding how uncertainty affects investment as it complements other mechanisms highlighted in the literature, such as investment frictions or financial imperfections (e.g. Aghion et al., 2010; Alessandria et al., 2015; Arellano et al., 2019; Bloom, 2013; Handley et al., 2022; Martin et al., 2023; Merga, 2020; Pindyck, 1982).

Layout. In Section 2, I start with a simplified model to highlight the mechanism's intuition. Section 3 presents the data. Sections 4 use micro-level data to test the model's main assumptions. Section 5 presents the cross-country conditional correlations. Section 6 introduces the general equilibrium model, and Section 7 introduces the quantitative predictions of the baseline model and its version without the proposed mechanism. Section 8 concludes.

2 The mechanism in a simple example

This section highlights the mechanism's intuition in a simple example model. It starts by showing how the firms' revenue function curvature influences firm-level volatility effects on exports. Then, it illustrates how the curvature of the revenue function is affected by assumptions about the price elasticity of demand.

The model consists of a continuum of firms that solve a two-period problem. Firms start with a certain amount of customer capital, A. Their export status, m, is determined by a Bernoulli random variable (with probability ι). They produce the variety quantities, q, using a linear technology in labor l and productivity z drawn from a continuous distribution, $F(z)$, with a standard deviation (s.d.) denoted as σ_z. Firms' investment to expand their customer capital is sunk, and it takes place before both the productivity shock z_i and the export status are known. The firm's demand is given by

$$q(A,p,Q^f) = A^{\alpha}\hat{q}(p)Q^f, \tag{1}$$

A^{α} is the demand shifter that depends on firms' customer capital, $\hat{q}(p) := q(1,p)$ is the static component of demand depending on price, p, and Q^f is foreign economy total expenditure - which is constant for now. The firm's static problem is to choose a price to maximize its profits, $\pi(A,z,m)$, after observing z and m. Firms' optimal profits can be rewritten as

$$\pi(A,z,m) = A^{\alpha}\hat{\pi}(z,m),$$

where $\hat{\pi}(z,m) := \pi(1,z,m)$. When $m = 0$, then $\hat{\pi}(z,0) = 0$. The firms' dynamic problem is:[4]

$$\max_{A' \in [0;\infty)} A^{\alpha}\hat{\pi}(z,m) - wA' + \beta \mathbb{E}_{z'}\left\{A'^{\alpha}\hat{\pi}(z',m')|_{z=z}\right\} \tag{2}$$

If firms decide to invest, tomorrow's customer capital is given by

$$A'(z) = \left\{\frac{\alpha \iota \beta}{w}\mathbb{E}_{z'}\left\{\hat{\pi}(z',1)|_{z=z}\right\}\right\}^{\frac{1}{1-\alpha}} \tag{3}$$

Note how the curvature of profits with respect to firm productivity determines the impact of firm-level volatility on investment in equation (3). A mean-preserving spread in $F(z)$ increases the likelihood of both better and worse productivity outcomes. When profits are concave, the expected profit reductions from worse outcomes outweigh gains from better ones, lowering the investment's expected return. The opposite holds true when profits are convex.

Simplify the analysis assuming that firms' productivity follows an independently and identically distributed (iid) process. Next period's total exports are given by:

$$Exp = A^{\alpha}\int p(z)\hat{q}(z)dF(z)$$

where $A'(z) = A \ \forall \ z$ since $z \sim iid$. Define $G(z)$ as a mean-preserving spread of $F(z)$, and denote variables x_G as any variable x derived under distribution $G(z)$. We can write the log export ratio

[4]Firms use labor to invest in customer capital, which fully depreciates in the last period.

between a country with low and high firm-level volatility as,

$$\ln\left(\frac{Exp_G}{Exp}\right) = \underbrace{\ln\left(\frac{A_G^\alpha}{A^\alpha}\right)}_{\text{dynamic response}} + \underbrace{\ln\left(\frac{\int p(z)\hat{q}(z)dG(z)}{\int p(z)\hat{q}(z)dF(z)}\right)}_{\text{static response}} \tag{4}$$

The total export response to increased firm-level volatility depends on: (1) A dynamic response, capturing how increased uncertainty impacts firms' investment in customer capital; and (2) the static response, reflecting shifts in total sales due to changes in the productivity distribution.

Lemma 1. *If the production function is linear in inputs and the curvature of the revenue function is concave (convex) regarding firms' productivity, then a mean-preserving spread over firms' productivity reduces (increases) total exports.*

Proof: *See appendix A*

Lemma 1 results from two effects previously mentioned: the dynamic consequence of higher volatility on expected returns, equation (3), related to Proposition 1 below), and a static response. The latter reflects that under a mean-preserving spread, if revenues are concave, sales gains from positively affected firms are offset by losses in negatively affected ones; the converse holds for the convex case. As I will show later, this concavity can arise from frictions that induce price elasticity to decrease with a firm's productivity. Intuitively, in this case, more productive firms increase markups, which limits resource reallocation and sales growth to these firms—unlike the constant elasticity case where reallocation is strengthened.

Volatility also has implications for exporters' growth. Proposition 1 shows that exporters' growth is differentially impacted by the volatility of idiosyncratic productivity, depending on the underlying shape of their revenue function. I revisit this implication and test it in the empirical section.

Proposition 1. *Under monopolistic competition and linear production function, if the revenue function is continuous and concave (convex) on firms' productivity, then a mean-preserving spread over the firms' idiosyncratic productivity shocks reduces (increases) exporters' growth.*

Proof: *See appendix A*

Now I turn to show how volatility differences can affect estimated export costs under a misspecified model. Lemma 1 implies that revenue function misspecification biases trade determinant estimates. For example, assuming convexity when revenue is concave spuriously predicts a positive volatility-export relationship. To match the data, the misspecified model requires over-estimating the export costs as volatility increases. Ignoring dynamic export decisions similarly biases results as shown in equation (4).

Proposition 2. *Under monopolistic competition and linear production function, if the revenue function is continuous and miss-specified, assuming convexity instead of concavity, the convex model will over-estimate the export costs for an economy with a mean-preserving spread on firms' productivity.*

Proof: *See appendix A*

Let's now explore how demand price elasticity assumptions impact the revenue function's curvature. To gain some economic intuition on this result, recall that productivity increases revenue via lower prices and higher quantities sold. When price elasticity is constant, productivity fully passes through to lower prices, leading to a more than proportional increase in quantity demanded and revenues.[5] However, if the price elasticity falls with prices, firms moderate price cuts, increasing markups, and weakening the direct price effect. Simultaneously, demand becomes less price-sensitive, dampening the quantity response. This can lead to a concave revenue-productivity relationship depending on the elasticity's responsiveness to prices. This result is similar to the specific case shown in Klenow et al. (2016).

Proposition 3. *Under monopolistic competition and linear production function, if the price elasticity is sensitive enough to firms' prices, then revenues become a concave function of firms' productivity.*

Proof: *See appendix A*

Proposition 3 explains why the models with constant elasticity fail to generate a negative relationship between firm-level volatility and total exports. They generate a convex revenue function in firms' productivity. Crucially, the proposition also implies that the presence of variable price elasticity is insufficient to guarantee a concave revenue function. The model's capacity to generate concave revenues depends on the degree of price elasticity variability. Therefore, in the quantitative section, I estimate the key parameters governing this variability using indirect inference.

[5] If price elasticity is higher than one.

In the following sections, I use micro-level data to test the models' main assumptions: the co-existence of variable markups and new exporters' dynamics. After confirming the existence of both channels, I test the model's key predictions. Then, I extend the model to a general equilibrium framework with endogenous exit decisions and persistent productivity shocks to show the quantitative relevance of the proposed mechanism.

3 Data

In this section, I discuss the data. To document the aggregate facts, I use two main data sources: the Enterprise Survey from the World Bank and the Trade and Production Database (TradeProd) from CEPII. [6]

Cross-country data. The TradeProd database offers several key advantages for my analysis. It covers 162 countries and nine industrial sectors over the period 1966-2018. Critically, it reports both domestic and foreign sales, facilitating the estimation of export costs. Furthermore, it also allows me to exploit the use of a border dummy to quantify the differential impact of firm-level volatility on export relative to domestic sales, which I do in the appendix. The database also includes pertinent control variables, which I expand by merging with the CEPII Gravity database detailed in Conte et al. (2022). This allows me to estimate bilateral and aggregate export costs, conditional on important country characteristics.

However, a database limitation is its sectoral scope. It includes only nine relatively aggregated industrial sectors, but given the proposed mechanism, this is not a primary concern, as while the forces I examine operate across all sectors, they are likely more salient for industrial ones.[7]

To obtain the firm-level statistics for the cross-country analysis, I use the World Bank Enterprise Surveys (WBES) for the period 2006 to 2024. This dataset comprises nationally representative firm-level surveys across over 160 economies. A key advantage is its explicit design for cross-country comparability.[8] Additionally, the database provides specific firm and country weights, enabling me to compute within-country representative estimates. These weights are employed for all reported statistics throughout the paper unless otherwise stated.

[6]For details regarding the (TradeProd) database see (de Sousa et al., 2012; Mayer et al., 2023).
[7]For more details see Mayer et al. (2023)
[8]Details regarding the sample methodology can be found in the WBES sampling note available here.

There are two main limitations with the WBES database. One is the uneven data availability across countries and time. However, this missing data is unlikely to introduce significant biases, as the reasons for its temporal and geographic dispersion are unrelated to this paper's question. Another potential concern is the inherent potential bias towards the formal sector of the economy. Since the focus is on exporters, who are larger and operate within the formal sector, the last concern is unlikely to be relevant.

Firm-level data. For the firm-level data and model estimation, I use two primary data sources: (1) Administrative data from Colombian customs and (2) Administrative data from "Superintendencia de Sociedades" from Colombia containing the firm's balance sheet information. The first data set reports exports of each firm at the 8-digit product level for each destination and period. The data is monthly and provides information on the quantities shipped and the value of the shipment in Colombian pesos and U.S. dollars over the period 2006-2019. I aggregate export flows at the firm-product-destination level yearly to avoid the usual problems with lumpiness in international trade.

I merge the custom data with firm-level data from "Superintendencia de Sociedades", which reports the variables from firms' balance sheet information. This dataset provides information on firms' total income, operational income, operational cost, total costs, profits, and operational profits. These variables are in nominal Colombian Pesos, which I deflate with the production price index when needed. The data sets cover a sub-sample of 20,000 firms a year between 2006 and 2015. These firms are the most prominent, representing around 90% of total value-added in the country.[9]

4 Firm-level facts

This section presents three firm-level facts that support the underlying assumptions and predictions of the simplified model described in Section 2.

[9]The sample is skewed towards larger firms. However, since the paper focuses on exporters' behavior, this alleviates this concern as the largest firms in the economy are the ones that are exporters, and exports are highly concentrated among larger firms.

4.1 Markup responses and firm heterogeneity

Estimation. Assuming that firms' i prices, p, when selling product l and destination d, are set in foreign currency, prices are given by the following equation:

$$p_{i,d,l,t} = \mu_{i,d,l,t} \frac{Mc_{i,d,l,t}}{e_{d,t}}$$

where $e_{d,t}$ is the bilateral exchange rate, $\mu_{i,d,l,t}$ is the markup, and $Mc_{i,d,l,t}$ marginal costs in domestic currency.

If, as generally assumed, firm i's marginal cost is the product of two components—a production cost ($Mc^{a}_{i,l,t}$) common to all destinations, and a product-destination-specific selling cost ($Mc^{b}_{l,d,t}$)—then markup responses can be recovered by exploiting price variations at the firm-product-time and product-destination-time level, as follows:

$$\frac{\partial \ln p_{i,d,l,t}}{\partial \ln e_{d,t}} = \frac{\partial \ln \mu_{i,d,l,t}}{\partial \ln e_{d,t}} + \frac{\partial \ln Mc^{1}_{i,l,t}}{\partial \ln e_{d,t}} + \frac{\partial \ln Mc^{2}_{l,d,t}}{\partial \ln e_{d,t}} - 1$$

Given the previous result, and consistent with the model to be used later, firms' markup responses to shocks are contingent on their relative productivity to the market they serve, which can be proxied by firms' market share (Arkolakis et al., 2017); these responses can then be pin down by estimating the following equation:

$$\begin{aligned}\Delta p_{i,d,l,t} = {} & \beta_1 \Delta e_{i,d,l,t} \times \text{exp. share}_{i,d,l,t-1} + \boldsymbol{\beta}\text{exp. share}_{i,d,l,t-1} \times \boldsymbol{X}_{d,l,t} \\ & + \theta_{i,l,t} + \gamma^{2}_{i,l,d} + \gamma^{3}_{l,d,t} + e_{i,d,l,t}\end{aligned} \tag{5}$$

where Δ denotes log differences of the variables over a year, and $\boldsymbol{X}_{d,l,t}$ represents a matrix with unit vector, log changes in destination import prices, Colombian aggregate export prices, and the destination's real GDP. $\theta_{i,l,t}$, $\gamma^{2}_{i,l,d}$ and $\gamma^{3}_{l,d,t}$ denotes firm-product-time, firm-product-destination, and product-destination-time fixed effects. Hence, β_1, the coefficient of interest, captures the differential markup responses to exchange rate movements driven by firms' differences in destination market share.[10]

However, estimating (5) by OLS might generate biased estimates for β_1. This is because when bilateral exchange rates are driven by changes in destination markets conditions, they can affect

[10]This estimation procedure cannot be used to estimate the level of markup, as it only captures the markup responses to shocks depending on exporters' relative size.

firms' market share and thus markups, but not through cost changes, thereby biasing the β_1 estimates. To circumvent this, I instrument bilateral exchange rate variation (interacted with firms' sales shares in 2007) using remittance flows from third countries to Colombia ($remittances_{d,t}$) thereby mitigating concerns about exchange rate fluctuations driven by destination-specific shocks. The first stage is then given by

$$\Delta e_{i,d,l,t} \times \text{exp. share}_{i,d,l,t-1} = \Delta remittances_{d,t} \times \text{exp. share}_{i,d,l,07} + \\ + \beta \text{exp. share}_{i,d,l,t-1} \times \mathbf{X} + \theta_{i,l,t} + \gamma^2_{i,l,d} + \gamma^3_{l,d,t} + \varepsilon_{i,d,l,t} \tag{6}$$

Firm-level fact 1: Markup response increases with the firm's market share. Table 1 presents the estimation results. Panel 1 shows the estimates for the first stage. The instrument is strong enough, as the F-statistic range is between 80 (column 2) and 101 (column 4). Panel 2 shows that the markup response to changes in firms' marginal cost is increasing in the exporter's market share. Specifically, exporters with a one percentage point higher market share increase their markups between 0.82% and 0.65% in response to a 1% decrease in their marginal cost (columns 2 and 4, respectively).

Panel 3 provides insights into the instrument's soundness as it presents a similar estimate but uses quantities as the dependent variable. The OLS estimates—columns 1 and 3—predict a quantity change inconsistent with the predicted price changes in panel 2, while the IV results—columns 2 and 4—show results consistent with the predicted price changes in panel 1. The IV estimates suggest an average price elasticity ranging between two and five, consistent with the literature.

Appendix B.2 discusses the IV strategy in more detail and presents several robustness results as shown in Table A.7. The results are robust to dropping the firm-destination-product fixed effects, conditioning only to exporters that continue exporting the following year, and repeating the analyses conditional on fixing firms' market share after 2012.[11].

4.2 Exporters' growth and firm-level volatility

I now turn to test the model's assumption regarding new exporters' dynamics and its implications for exporters' growth under uncertainty highlighted in section 2.

[11] I use 2012, since after this year the Colombian economy recovered from the global financial crisis

Table 1: Heterogeneous Markup Responses

	(OLS)	(IV)	(OLS)	(IV)
Panel 1: First Stage				
	Dependent variable: Δex. rate$_{d,t}$ $\times$ $share_{i,l,d,t-1}$			
Δremittances$_{\neq d,t}$ $\times$ $share_{i,l,d,07}$	-	0.28***	-	0.29***
	-	[0.03]	-	[0.03]
Panel 2: Second Stage (Prices)				
	Dependent variable: $\Delta \log p_{i,l,d,t}$			
Δexchange rate$_{d,t}$ $\times$ $share_{i,l,d,t-1}$	0.11	0.82***	0.09	0.65**
	[0.08]	[0.29]	[0.10]	[0.29]
Panel 3: Second Stage (Quantities)				
	Dependent variable: $\Delta \log q_{i,l,d,t}$			
Δexchange rate$_{d,t}$ $\times$ $share_{i,l,d,t-1}$	0.77***	-3.21***	0.25	-2.09***
	[0.21]	[0.70]	[0.21]	[0.62]
Observations	62,357	62,357	58,781	58,781
F-statistic		80.68		101.81
Firm-product-Destination FE			✓	✓
Controls $\times$ $share_{i,l,d,t}$	Agg. prices	Agg. prices	All	All

Note: All cases includes Destination-product-time and Firm-product-time fixed effects. Panel 1 shows the first-stage results. Panel 2 shows the results using the log difference of unit values over a year. Panel 3 shows the estimated results for quantities exported. Exporter age denotes the minimum age of an exporter in the sample. Controls $\times$ $share_{i,l,d,t}$ denotes the addition of controls of firms' sales share among total Colombian exports and its intersection with the log change of real GDP, Colombia export price to that destination, and the import price index. "Agg. prices" denotes when only aggregate price changes are used, and "All" denotes the case, including GDP changes. Standard errors in brackets. Error cluster at the destination country. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Estimation. The evolution of customer capital is unobservable. However, according to the demand equation (1) in Section 2, it can be identified by removing relative price changes from exporters' export intensity evolution over their life cycles:[12]

$$\Delta \text{exp int}_{i,l,d,t} = \alpha \overbrace{\Delta A_{i,l,d,t}}^{\Delta\text{customer capital}} + \Delta \overbrace{\frac{\hat{q}(p_{i,l,d,t})}{\hat{q}(p_{i,l,\text{dom},t})}}^{\Delta\text{relative price}}$$

Hence the drivers of exporters' growth can be asses by estimating:

[12]Note that here I am abstracting from the market aggregate variables changes, since the destination time fixed effects take care of them.

$$\Delta_{h(i,l,d)} y_{i,d,l,t} = \sum_{h=0} \beta_h \mathbb{I}^h_{\{\text{age}_{(i,l,d)}=h\}} + \beta_2 \ln p_{i,d,l,t} + \gamma^a_{i,l,t} + \gamma^b_{d,l,t} + \gamma^c_{\text{cohorts}} + \varepsilon_{i,d,l,t} \; ; \tag{7}$$

where $\Delta_h y$ is the log difference of the dependent variable at year t relative to the value when the exporter entered the market, $h(i,l,d)$ years ago.[13] The dependent variable is firms' i export intensity, defined as product l sales to destination d over total domestic sales. The key regressor is an age dummy, $\mathbb{I}^h_{\{\text{age}=h\}}$, indicating h years of continuous export of product l to destination d. The specification controls for product prices ($\ln p_{i,d,l,t}$), firm-product-year fixed effects ($\gamma^a_{i,l,t}$), product-destination-year fixed effects ($\gamma^b_{d,l,t}$), and entry cohort fixed effects ($\gamma^c_{\text{cohorts}}$).[14] These fixed effects absorb common sales variation across markets for a given firm-product-year and common variation across exporters within a destination-product-year. Hence, the vector $\{\beta_h\}^6_{h=1}$ estimates the average cumulative change in export intensity relative to each firm's entry value, conditional on firm-level prices and controlling for any product-destination variation. Price dynamics are then estimated using the same specification but omitting the price control.

Firm-level fact 2: New exporters grow by shifting their demand curve. Panel (a) of Figure 1 presents the estimates of the evolution of exporters' export intensity, conditional on prices, over their life cycle after entering a new market. Five years after entry, conditional on survival, export intensity grows around 40%. Panel (b) shows the relative price evolution over exporters' life cycle. Prices tend to be flat on average over the exporters' life cycle in a specific market. These two results imply that export intensity expansion into foreign markets is driven by shifts in exporters' intercept of the demand as in Fitzgerald. et al. (2024) and Steinberg (2023). The model will capture these dynamics through changes in customer capital.[15]

Firm's exposure to volatility and exporters' growth. I now explore how firm-level volatility relates to exporters' growth over their life cycle. To construct the firm-level volatility measures, I use within-firm annual changes in domestic sales. Specifically, I isolate the idiosyncratic changes by purging out aggregate and industry factors as follows

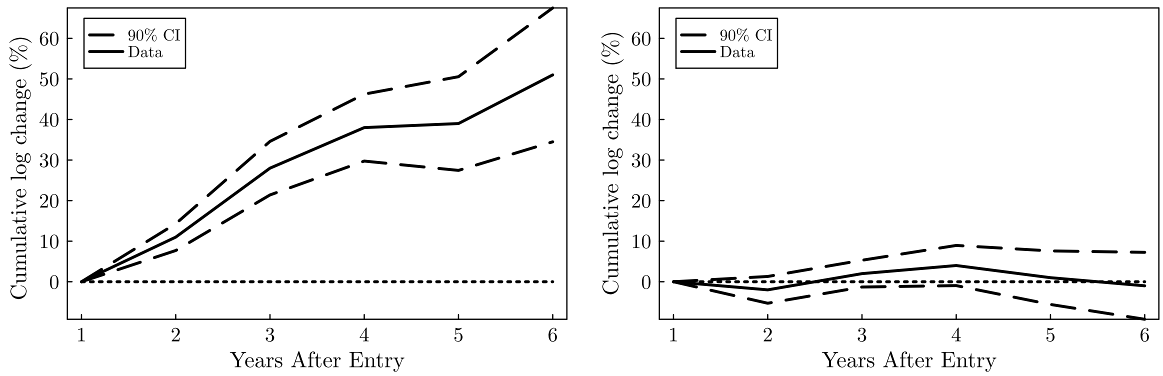
$$\Delta \text{dom. sales}_{i,j(i),t} = \gamma_{j(i),t} + e_{i,j(i),t}$$

[13] A new exporter is defined as an exporter that did not export product l, to destination d in the previous 2 years

[14] Prices are proxied by unit values.

[15] The estimated coefficients used in Figure 1 are presented in column 10 of Table A.5 in the appendix, together with other robustness tests.

Figure 1: New Exporters' Dynamics



(a) Export intensity dynamics

(b) Price dynamics

Note: Panel (a) shows the estimated log cumulative change in export intensity relative to total sales, relative to firms' first year of export to the market. Panel (b) shows the same, but for price changes. A market is a six-digit product-destination combination. Both estimates include firm-product-time, destination-time, and cohort fixed effects. Results in Panel (a) are presented in column 7 of Table A.5, and results from Panel (b) are from column 10 of the same table. Firms in the sample are exporters that continuously export to each market, and a new exporter is a firm that exports at time t after at least three years of not exporting to the market. Standard errors in brackets. Error cluster at the firm level.

where $\Delta\text{dom. sales}_{i,j(i),c,t}$ is the log change in domestic sales of firm i, whose main industry is $j(i)$ during year t. $\gamma_{j(i),t}$ denotes industry-year fixed effects, so that $e_{i,j(i),t}$ can be interpreted as pure idiosyncratic firm-level changes in domestic sales (Di Giovanni et al., 2024). Once the $e_{i,j(i),t}$ are estimated, I use a leave-one-out strategy to compute the firm's exposure to domestic firm-level volatility ($\sigma_{i,t}$). Firm's exposure to domestic firm-level volatility is computed as the average cross-sectional s.d. of these idiosyncratic shocks across other firms than i in the same industry at time t. This leave-one-out strategy ensures results are not driven by first-moment shocks affecting firm i, nor by aggregate or industry shocks. See Appendix B.2 for more details and several robustness checks.

Estimation. To assess how domestic firm-level volatility relates to exporters' life-cycle, I estimate the same equation as in (7), expanded with firms' volatility measure as follows:

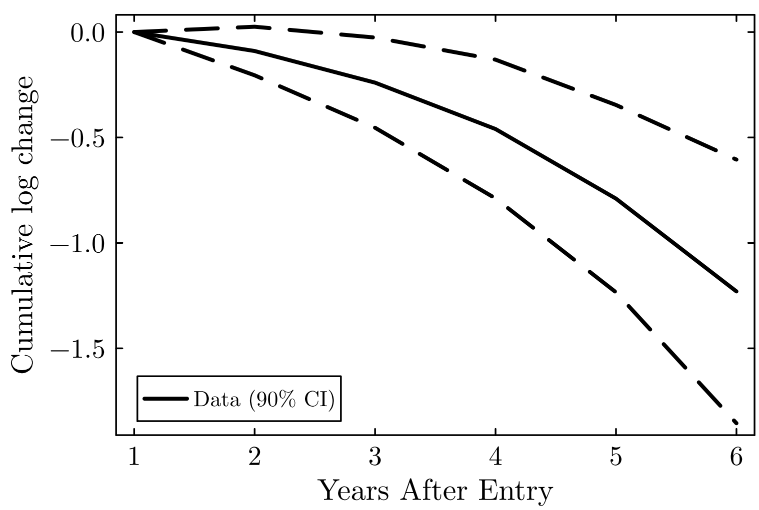
$$\begin{aligned}\Delta_{h(i,l,d)} y_{i,d,l,t} = \sum_{h=0} \beta_1^h \mathbb{I}_{\{\text{age}_{i,l,d}=h\}} \ln \sigma_{i,t} + \sum_{h=0} \beta_2^h \mathbb{I}_{\{\text{age}=h\}} + \beta_3 \ln \sigma_{i,t} \\ + \gamma^a_{i,l,t} + \gamma^b_{d,t} + \gamma^c_{cohort_i,l,d,t} + \beta_4 \ln p_{i,d,l,t} + e_{i,l,d,t}\end{aligned} \tag{8}$$

All variables and fixed effects are the same as before. But now, the coefficients of interest is the vector of $\{\beta_1^h\}_{h=1}^{6}$ which shows the differential cumulative export performance of firms over their life cycles following a 1% increase in domestic firm-level sales volatility measures.

Firm-level fact 3: Domestic firm volatility slows new exporter growth. Figure 2 presents the estimates for $\{\beta_1^h\}_{h=1}^6$. The estimates show that as firms' exposure volatility increases, exporters reduce their cumulative growth; specifically, a 1% increase in volatility reduces new exporters' export intensity cumulative growth by more than one log percentage point six years after entry. This result is consistent with Proposition 1.

The estimated coefficients used in Figure 2 are presented in column (6) of Table A.6 in the appendix, together with other robustness tests. The appendix B.2, presents several robustness checks for both firm-level facts. For example, using less strict fixed effects, changing exporters' minimum tenure in the export market, changing the dependent variable, and using different measures of exposure to domestic firm-level volatility. All results remain unchanged.

Figure 2: New Exporters Growth and Firm-level Volatility.



Note: The data results show the estimated coefficient for $\{\beta_1^h\}_{h=0}^6$, which captures how firms' export intensity changes after a 1% increase in a firm's exposure to volatility. The estimated coefficients together with additional robustness are presented in column 6 of Appendix Table A.6.

5 Aggregate Facts

This section presents two novel cross-sectional findings concerning firm-level volatility, export costs, and development. Since these facts are jointly determined in equilibrium, they serve as conditional correlations for testing the general equilibrium model's predictions in subsequent sections. I first detail the estimation of export costs and idiosyncratic firm-level volatility for each country. I then revisit a known stylized fact from the literature before presenting the two novel findings.

Export costs measurement. As I explained before I focus on manufacturing data.[16] To estimate countries' export costs I follow Waugh (2010) assuming that bilateral trade costs from i to j at time t, $d_{ij,t}$, are given by:

$$\ln d_{ij,t} = \ln \text{exp cost}_{i,t} + \ln \widehat{d}_{ij,t} \ ;$$

$\widehat{d}_{ij,t}$ is the "pure" bilateral trade costs, usually associate with distance and to be detailed later; and exp cost$_{i,t}$ denotes the common export country i faces when exporting to any destination.

Define $X_t^{i,j}$ as country j's expenditure share on goods from country i, and $\lambda_t^{i,j} \equiv \frac{X_t^{i,j}}{X_t^{j,j}}$. The usual argument for models with gravity structure (Eaton et al., 2002; Waugh, 2010) implies:

$$\ln \frac{X_t^{i,j}}{X_t^{j,j}} = \mathrm{S}_{i,t} - \mathrm{S}_{j,t} - \theta \left(\text{exp cost}_{i,t} + d_{ij,t} \right)$$

$\mathrm{S}_{i,t}$, captures the multilateral resistance term of country i (Waugh, 2010). To estimate export costs, first, I estimate equation (9) via Poisson Pseudo-Maximum Likelihood (PPML), as in Silva et al. (2006), which allows for consistent measurement of multilateral resistance terms (Fally, 2015).

$$\lambda_t^{i,j} = \mathbf{e}^{(\text{imp. FE}_{j,t} + \text{exp. FE}_{i,t} + \boldsymbol{\beta} \mathbf{y}_{ijt} + \varepsilon_{ijt})} \tag{9}$$

Note that the import fixed effects pins down $\mathrm{S}_{j,t}$, while the exporter fixed effects pins down $\mathrm{S}_{j,t} - \text{exp cost}_{j,t}$.

I perform the estimation separately for each year. The vector $\boldsymbol{y}_{cdt}$, captures the pure bilateral trade costs $\widehat{d}_{ij,t}$, for which includes standard gravity controls for each bilateral country pair: (1) log distance between most populated cities (in km); (2) UN diplomatic disagreement score; and indicator variables for (3) contiguous borders; (4) common official or primary language; (5) for when at least 9% of population share a common language; (6) for past colonial ties; and (7) for when a free trade agreement is in place.[17]

In a second step I compute export costs using the estimated exporter and importer fixed effects:

$$\widehat{\text{exp costs}}_{i,t} = \frac{\text{imp. FE}_{i,t} - \text{exp. FE}_{i,t}}{\theta} = \frac{\mathrm{S}_{i,t} \overbrace{- \mathrm{S}_{i,t} + \theta \text{exp costs}_{i,t}}^{= -\text{exp. FE}_{i,t}}}{\theta} \ ;$$

[16] Results hold similar when allowing all industries.

[17] All these bilateral control variables come from CEPII Gravity database, detailed in Conte et al. (2022).

θ is exogenously fixed at 2.5 to be consistent with the quantitative model coming later.[18] Note that by construction, the estimated export costs are independent of countries' productivity, aggregate prices, and foreign demand encompassed in the multilateral resistance terms $S_{i,t}$. Hence, export costs are by construction independent from these factors (Eaton et al., 2002; Waugh, 2010).

Firm-level sales volatility measurement. To measure the country firm-level volatility, I proceed similarly as before. I use within-firm annual domestic sales changes from the World Bank Enterprise Surveys. To be consistent with the trade data, in the baseline case I focus on manufacturing only. I isolate the idiosyncratic changes by purging out aggregate and industry factors as follows

$$\Delta \text{domestic sales}_{i,j(i),c,t} = \gamma_{j(i),c,t} + e_{i,j(i),c,t}$$

$\Delta \text{domestic sales}_{i,j(i),c,t}$ denotes the percentage change in variable of firm i, whose main industry, $j(i)$, in-country origin c, during year t. $\gamma_{j(i),c,t}$ denotes country-industry-year fixed effects. Hence, $e_{i,c,t}$ can be interpreted as a pure firm-level shock to domestic sales as before. This approach ensures that $e_{i,j(i),c,t}$ is not directly driven by firm-level foreign demand, industry, or aggregate shocks. Additionally, by weighting firm sales changes equally, the volatility measures are not driven by granular firms.[19] Once the $e_{i,j(i),c,t}$ are estimated, the country's c firm-level volatility in year t, $\sigma_{c,t}$, is defined as the cross-sectional observed s.d. of $e_{i,j(i),c,t}$.

Estimation. With the volatility and export costs measures at hand, I estimate the following equation:

$$\widehat{\text{exp costs}}_{c,t} = \beta_0 + \beta_1 \ln \sigma_{c,t} + \beta_2 \ln \frac{GDP_{c,t}}{L_{c,t}} + \beta_3 \ln \text{fin. fric}_{c,t} + \boldsymbol{\beta}_4 h_{c,t} + \gamma_t + e_{c,t} \ ; \qquad (10)$$

the two main coefficients of interest are β_1 and β_2. The former captures the percent change in the estimated export cost after a one percent change in the firm-level volatility, σ_c. The latter captures the same relationship, but between the export costs and the GDP per capita of the country.[20] The additional control, fin. $\text{fric}_{c,t}$, denotes the share of firms declaring access to financial markets as an impediment to growth. The vector $h_{c,t}$, denotes a control for firms' entry costs -

[18] A large body of literature has estimated the elasticity to range from one to five Boehm et al. (2023); Simonovska et al. (2014).

[19] Granular firms can be important drivers of business cycles in some countries (Di Giovanni et al., 2024)

[20] I use USD GDP adjusted for purchasing power parity in 2011 dollars from Penn World Tables, divided by that year's total population to construct the GDP per capita

proxy by the numbers of procedures to register a business- and a set of indicator capturing if the country belongs to the European union, the world trade organization, is a GATT membership, and a categorical for the origins of the current the legal system type. These controls, as the bilateral ones, come from the CEPII Gravity database. γ_t denotes year fixed effects.

Table 2 presents the estimation results for equation (10). Columns 1-3 report findings with export costs as the dependent variable, while Column 4 uses the log of GDP per capita. This last column reveals a negative relationship between firm-level volatility and GDP per capita, consistent with established findings on growth and macroeconomic volatility (Aghion et al., 2010; Ramey et al., 1995).

Table 2: Firm-level Volatility, Development and Exports Costs

	Export costs			GDP per capita
	(1)	(2)	(3)	(4)
ln (GDP per capita)	-0.699***	-0.466***	-0.447***	
	(0.125)	(0.109)	(0.114)	
ln (Firm volatility)		0.575***	0.577***	-0.381***
		(0.102)	(0.101)	(0.120)
ln (Financial frictions)			0.086	-0.316**
			(0.126)	(0.122)
N	126	126	126	126
Adjsuted R^2	0.448	0.591	0.590	0.441
Year FE	✓	✓	✓	✓
Controls	✓	✓	✓	✓

Note: Table reports estimates of equation (10). Export costs are estimated using PPML, following a gravity specification similar to Waugh (2010) (equation 9). Annual trade flows and firm-level volatility used are for manufacturing. Controls include financial access (WBES database), entry costs (number of procedures), and dummies for EU, WTO, GATT membership, and legal origin. Standard errors clustered at the origin country level are in parentheses. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Aggregate Fact 1: Export costs decline with GDP per capita. Column (1) of Table 2 shows that a 1 percent increase in a country's GDP per capita is associated with a 0.7 percent decrease in its average export costs. This significant negative relationship persists even after controlling for aggregate firm-level volatility or the level of financial frictions (columns 2 and 3). The financial friction variable is not statistically significant in explaining export costs differences (Column 3), consistent with the findings in Leibovici (2021).

Fact 1 restates well-established findings in the literature (Blum et al., 2019; de Sousa et al., 2012; Waugh, 2010). Given that export cost estimates account for cross-country differences in productivity, prices, and bilateral trade costs (Eaton et al., 2002; Fally, 2015; Waugh, 2010), the explanation for this relationship lies in factors that negatively correlate with development while positively correlating with estimated export costs..

Aggregate Fact 2: Exports costs increase with firm-level sales volatility. Columns (2) and (3) of Table 2 show that there exists a positive relationship between a country's export costs and its firm-sales volatility, even conditional on the country's level of financial development or GDP per capita. The estimates show that a country with a 1 percent higher firm-level volatility will face, on average, 0.57 percent higher export costs. In terms of trade flows, this translates to a 1.4% log points reduction in exports to each destination on average. Put it differently, moving from the median to the first quartile of the cross-sectional distribution of firms' sales volatility is associated with a decrease in export costs of around 44%, an increase in exports of 111%. This result is consistent with Proposition 2.

Aggregate Fact 3: firm-level volatility dampens the export cost-GDP per capita relationship. Conditional on the firm-level sales volatility, the estimated relationship between exports and GDP per capita decreases by approximately 33% compared to the baseline (column 1). This reduction is statistically significant.[21] Furthermore, the adjusted R^2 improves by nearly 32% when firm-level volatility is included.

To address potential biases, I performed several robustness checks discussed in Appendix B.1. These include using different sample restrictions, variables, or statistics to compute firm-level volatility. I also test whether the results are robust when using other methodological procedures. Specifically, I estimate the relationship between total sales (domestic and exports) and firm-level volatility, GDP per capita, and financial development, using a border dummy to identify the differential relationship that domestic volatility has on exports relative to domestic sales. The findings are robust to these alternative specifications.

Clearly, these aggregate facts reveal conditional correlations that result from equilibrium relationships, yet nonetheless provide direct, suggestive evidence that firm-level volatility can be

[21] The two-sided p-value for the null hypothesis of no difference is 0.036; and the p-value for the estimate in column 2 being smaller or equal to that in column 1 is 0.018

a relevant factor in explaining cross-country differences in export costs and GDP per capita. Because of this, in the following sections, I employ a general equilibrium model to investigate whether changes in the volatility of firms' productivity can generate these patterns, once the firm-level facts previously discussed are taken into account.

6 The model

This section describes the small open general equilibrium model used in the next section to quantitatively assess the macroeconomic relevance of the proposed mechanism.

The economy consists of a continuum of firms producing intermediate goods, a representative firm producing a domestic bundle, a final good firm producing the consumption good, and a representative household. The final consumption goods and the domestic bundle producers operate in a competitive market. There are no aggregate shocks.

Domestic consumers. The representative consumer of this economy owns the firms and holds risk-free bonds in zero net supply, so trade is balanced. Every period, she observes her bond holdings, b, and the aggregate state of the economy $\mathbb{S}$, decides how much to consume and save, and provides labor inelastically, L^s. Her problem is given by:

$$V^c(b,\mathbb{S}) = \max_{b',C} u(C,L) + \beta \mathbb{E}\left\{V^c(b',\mathbb{S}')\right\}$$

s.t.

$$P^C C + b' = wL^s + \Pi^{dom} + \Pi^{exp} + r_t b'$$

The household problem determines the stochastic discount factor for the firm given by $\Lambda = \beta \frac{u_c(C',L)}{u_c(C,L)}$.

Final good production. The final consumption good is produced using a bundle of imported goods, M, and a bundle of domestic goods, D; these bundles are combined in the following way to produce the final good C,

$$\left(M^{\frac{\gamma-1}{\gamma}}\nu + (1-\nu)D^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \geq C \,, \tag{11}$$

where $(1-\nu)$ represents the home bias. The price of each of these bundles is given by P^m and P^D, respectively. P^m is, from now on, normalized to one. The final good firm chooses the amount of domestic and imported consumption bundles to solve

$$\min_{M,D} M + P^D D$$

subject to (11). The solution to this problem yields the following demand for the domestic bundle:

$$D = M\left(\frac{\nu}{1-\nu}\right)^{-\gamma}\left(P^D\right)^{-\gamma} \tag{12}$$

Domestic bundle. The production for the domestic bundle, D, uses intermediate differentiated goods and is given by the following condition

$$\int_{\omega\in\Omega^d} \Upsilon\left(\frac{q^d(\omega)}{D}\right) d\omega = 1, \tag{13}$$

as in Klenow et al. (2016), $\Upsilon(x)$ is given by

$$\Upsilon(x) = 1 + (\theta-1)e^{\frac{1}{\eta}}\eta^{\frac{\theta}{\eta}-1}\left(\Gamma(\frac{\theta}{\eta},\frac{1}{\eta}) - \Gamma(\frac{\theta}{\eta},\frac{x^{\frac{\eta}{\theta}}}{\eta})\right),\ \theta > 1;\ \eta > 0 \tag{14}$$

where $\Gamma(a,b)$ represents the incomplete gamma function, I call θ the price elasticity parameter, and η the super-elasticity parameter. As it will be clear later, conditional on θ, η shapes the firm's markup responses to changes in the intermediate good price. The producer of the domestic bundle observes intermediate good prices $\{p^d(\omega)\}_{\omega\in\Omega}$ and chooses the intermediate quantities $\{q^d(\omega)\}_{\omega\in\Omega}$ to solve the following problem

$$\min_{q(\omega)} \int_{\omega\in\Omega} p^d(\omega) q^d(\omega) d\omega,$$

subject to equations (14), and (13). The solution to this problem yields the following demand for variety ω,

$$\log q(\omega) = \frac{\theta}{\eta}\log\left(-\eta\log\left(\frac{p^d(\omega)}{p_c^d}\right)\right) + \log D \quad \text{if } p^d < p_c^d, \tag{15}$$

where p_c^d is the choke price for the domestic varieties in the economy - the maximum price at which the domestic bundle producer is willing to buy a variety - and is given by

$$p^c = e^{\frac{1}{\eta}}\frac{\theta-1}{\theta}\frac{P}{\tilde{D}}, \tag{16}$$

where P is the price index for the intermediate goods, defined as $P := \int_\Omega \frac{q(\omega)}{D}p(\omega)d\omega$, and $\tilde{D} := \int_\Omega \Upsilon'(\frac{q(\omega)}{D})\frac{q(\omega)}{D}d\omega$.[22]

[22]See Arkolakis et al. (2017) to see why when $\eta \to 0$ the model converges to CES, and $p^c \to \infty$.

Foreign consumer's problem. Intermediate firms can sell to a foreign importer. The importer takes aggregate foreign demand, Q^*, and foreign prices, P^*, as given.[23] The importer observes the prices of the intermediate goods and solves,

$$\min_{q^*(\omega)} \int_{\omega\in\Omega^*} p^*(\omega)q^*(\omega)d\omega$$

s.t.

$$\int_{\omega\in\Omega^*} A^{\alpha}(\omega)\Upsilon\left(\frac{q^*(\omega)}{A^{\alpha}(\omega)Q^*}\right)d\omega = 1,$$

where indirect utility function term $\Upsilon(x)$ is given by equation (14); $A(\omega)$ represents the customer capital that the exporter, producing variety ω, has when selling to this foreign market. α is the elasticity of customer capital to the demand intercept; as shows the following foreign demand function for each variety

$$\log q^*(A,p^*) = \frac{\theta}{\eta}\log\left(-\eta\log\left(\frac{p^*(\omega)}{p^{c*}}\right)\right) + \log A^{\alpha} + \log Q^* \quad \text{for } p^*(\omega) < p^{c*}, \tag{17}$$

note that $A(\omega)$ is a demand shifter, over which firms can invest and grow into the foreign market. As before, p^{c*} denotes the choke price of the foreign economy.[24]

Note that the equation shows how both cross-sectional variable markups are consistent with new exporters that grow by shifting the intercept of their demand. The evolution of $A(\omega)$ dictates the new exporters' dynamic, but the markup depends on the ratio between the firm's price and the choke price of the destination economy. Also, note that if $\alpha \to 0$, firms face no benefit from investing in customer capital; hence, there will be no new exporters' dynamics, and the model will behave as a model with static exporters' decisions.

Intermediate goods. Each intermediate firm produces a variety using a linear production function with time-varying labor productivity. The timing for the intermediate firms' decisions is as follows. At the beginning of time t, firm i observes its productivity z_i, drawn from a Markov process governed by the transition probability $f(z',z)$, and the foreign market level of customer capital A_i. It decides how much to sell to the domestic and foreign markets, sets the prices for each market, hires the workers, and produces - the static decision-. At the end of the period, it

[23]As the domestic economy is small, foreign aggregate price and foreign demand are assumed to be invariant to the condition of the domestic market.

[24]However, because the domestic economy is a small open economy, p^{c*} is assumed to be constant, unlike p^c, which is an equilibrium object.

decides how much to invest in the next period's customer capital to sell in the foreign market - the dynamic decision.[25] To be able to sell to foreign markets, firms need to pay the fixed cost, f_e, and they also face an iceberg cost, $\tau > 1$. Furthermore, the firms' customer capital depends on firms being present in the market; when a firm stops exporting, it loses the customer capital it accumulated.

Firms' static problem. The firm chooses the optimal price to maximize its operational profits, as in

$$\pi(z_i, A_i) = \max_{p_i, l_i} p_i^* q_i^*(A, p_i) - w l_i$$

subject to its production technology, $q_i^* = \frac{l_i z_i}{\tau}$, and demand equation (17).[26] Unlike the standard CES case, by choosing the price to maximize their profits, firms implicitly choose their price elasticity. By staring at equation (17), one can realize the firms' price elasticity, ξ is given by $\xi(p) = -\frac{\theta}{\eta \log(\frac{p}{p^{c*}})}$. The usual argument implies that firms' markups are given by,

$$\mu(p) = \frac{\theta}{\theta + \eta \log(\frac{p}{p^{c*}})} \quad \text{for all } p \leq p^{c*} \ ; \tag{18}$$

which are decreasing with firms' prices, and hence more productive firms charge higher markups, consistently with firm-level facts documented previously and the findings by Berman et al. (2012). [27]

Firms' dynamic problem. Denote with an apostrophe the variables next period, and by $\mathbb{S}$ the vector of aggregate state variables to simplify notation. Firms make two dynamic decisions: the exporting decision, denoted by m, and the investment decision to accumulate more customers, denoted by i_d. The decision of exporting or not in this model is a discrete decision given by $m \in \{0;1\}$. To invest i_d in customer capital, the amount of labor required is given by:

$$c(i_d, A) = i_d - \frac{\phi}{2}\left(\frac{i_d}{A_i}\right)^2 \tag{19}$$

Firms' customer capital is given by the following two components: a fixed minimum level of customer A^{min}, and the accumulated customer capital k_i. They relate to total customer capital as

[25] To simplify the computation burden and to be consistent with the previous empirical exercise, it is assumed that firms can reach all the available customers when selling to the domestic market.

[26] If $\alpha = 0$ and $\tau = 1$, the model becomes an static model with CES.

[27] The price elasticity equation and the markup equation imply boundaries for the optimal prices such that $\mu(p) \geq 1$, and $\xi(p) \geq 1$for all $p \leq p^{c*}$.

follows:

$$A_i = k_i + A^{min} \ , \tag{20}$$

which evolves according to the following law of motion,

$$k_i' = m\left(i_d + k_i(1-\delta)\right) \tag{21}$$

Firms can't sell their customer capital, and hence, they can't make negative investments, but when firms do not export ($m = 0$), they lose all the accumulated customer capital. Hence, tomorrow's customer capital is given by $A' = A^{min}$. The firm's dynamic problem is to solve

$$\begin{aligned} V(z_i, A_i, \mathbb{S}) = \max_{m\in\{0;1\};i_d\in[0;\infty)} & \pi^d(z_i,1) + m(\pi(z_i,A_i) - wf_e) - \\ & - wc(i_d, A_i) + \mathbb{E}\left[\Lambda(S)V(z_i', A_i', \mathbb{S}')\right] \end{aligned} \tag{22}$$

subject to (19), (20) and (21).

Firm's optimal dynamic behavior. The optimal customer capital the firm decides to have in the next period is given by:

$$\frac{\partial wc(i_d, A)}{\partial A'} \geq \Lambda \underbrace{(1 - Pr(z_i^{'*}|z_i))}_{\text{export probability}} \overbrace{\mathbb{E}_{z_i}\left\{\frac{\partial V(A', z')}{\partial A'} \ |z_i' > z_i^{'*}\right\}}^{\text{Expected marginal return if export}}$$

The condition holds with equality when firms decide to invest in customer capital. If so, firms equalize the investment marginal cost (left-hand side of the equation) to the expected marginal return on investment (right-hand side of the equation). The latter is determined by the expected probability of exporting the next period, denoted by $(1 - Pr(z_i^{'*}|z_i)$, and the marginal expected return of investment conditional on exporting, both negatively affected by uncertainty when profits are concave.

Firms will export if productivity is higher than the productivity threshold $z^*(A, \mathbb{S})$, given by:

$$\hat{\pi}(z_i^*, A) + \overbrace{\mathbb{E}_{z^*}\{\Lambda[V(A', z') - V((A^{min}, z')]\}}^{\text{Option value}} = w(f_e + c(i_d, A)),$$

The marginal firm is indifferent between staying in the export market or not if the sum of exports' operational profits from exports, plus the option value of not losing the customer capital it had accumulated, is equal to the investment cost plus the exporting fixed cost. The existence of the option value generates the well-known effects of hysteresis on international trade, whose absence can upward bias the negative effects of uncertainty on total trade, as its presence delays exit (see for example Alessandria et al., 2015; Merga, 2020).

6.1 Equilibrium

Let's now specify the conditions for equilibrium in this economy. Denote the firm productivity and customer capital joint distribution by $\Psi(z,A)$. Market clearing in the labor markets implies that inelastically supplied labor, L^s, equals labor demand determined by the sum of labor used for production, investment, and fixed costs. Total exports are given by

$$Exp = \int p^*(z,A)q^*(z,A)d\Psi(z,A),$$

because trade is balanced, nominal exports and imports are equal. Since $P^m = 1$, the demand for the domestic bundle is:

$$D = Imp\left(\frac{\nu}{1-\nu}\right)^{-\gamma}\left(P^d\right)^{-\gamma}$$

The price of the domestic bundle is given by $P^d = \int \frac{q(z,1)}{D}p(z)d\Psi(z,A)$, and the price of the consumption is given by P^C characterized by the usual price index for CES. The supply for the domestic bundle, D, is given by the following conditions,

$$\int \Upsilon\left(\frac{q(z,1)}{D}\right)d\Psi(z,A) = 1 \tag{23}$$

characterizing the equilibrium domestic choke price, p^c defined in equation (16). The evolution of the firm productivity and customer capital joint distribution, $\Psi(z,A)$, is given by:

$$H(z,A;\mathbb{S}_t) = \int f(z_t,z_{t-1})\phi(A_t,A_{t-1},z_{t-1};\mathbb{S}_{t-1})d\Psi(z_{t-1},A_{t-1}) \tag{24}$$

where $H(.)$ is the transition function for the measure of firms $\Psi_t = H(\mathbb{S}_{t-1})$. $\mathbb{S}_t$ denotes the aggregate state of the economy, and hence the measure of firms that transition from (A_{t-1},z_{t-1}) to (A_t,z_t) is denoted by $f(z_t,z_{t-1})\phi(A_t,A_{t-1},z_{t-1};\mathbb{S}_{t-1})$.

Given the initial measure Φ_0; an equilibrium consists of policy and value functions of intermediate goods firms $\{V(z,A,\mathbb{S}_t),A'(z,A,\mathbb{S}_t),q^s(z_t,\mathbb{S}_t),q^{*s}(z,A,\mathbb{S}_t),m(z,A,\mathbb{S}_t)\}$; of consumers $\{V^C(b,\mathbb{S}_t)\ ,b'(b_t,\mathbb{S}_t),\ C(b_t,\mathbb{S}_t)\}$; of final good producers $\{M(\mathbb{S}_t),\ D(\mathbb{S}_t)\}$; of domestic bundle producers $\{D(\mathbb{S}_t),\ q^d(\mathbb{S}_t)\}$; the price of the export and domestically sold intermediate goods $\{p^s(z,\mathbb{S}_t),p^{*s}(z,\mathbb{S}_t)\}$; the domestic choke price $\{p^c(\mathbb{S}_t)\}$; the price of labor units $\{w(\mathbb{S}_t)\}$; the price of the bonds $\{r(\mathbb{S}_t)\}$; the price of the consumption good and the domestic bundle, $\{P^c(\mathbb{S}_t), P^D(\mathbb{S}_t)\}$; and the evolution of the aggregate states Ψ_t governed by the function $H(\mathbb{S}_t)$, such that for all time (1) the policy and value function of intermediate good firms satisfy their optimal conditions, (2) domestic consumer decisions are optimal, (3), the final consumption producer and the domestic bundle producer decisions are optimal, (4) the bond market clears and trade is balanced, (5) labor and goods markets clear, and (6) the evolution of the measure of firms is consistent with the policy functions of the firms and consumers, and with their shocks.[28]

7 Quantitative Results

This section quantitatively assesses the model's ability to capture the firm-level facts and the documented relationships between firm-level volatility, total exports, and GDP per capita. I first discuss the model's parameterization, then present the quantitative predictions of the four models, evaluating the proposed mechanism's relevance.

7.1 Model calibration

Because the model is highly nonlinear, all parameters are set to match the moments together. However, some parameters have a clear empirical moment counterpart. The parameters values for each model are presented in Table 3. Two parameters are externally calibrated: the consumer's discount rate, β, and the Armington elasticity, γ, set to 0.98 and 2.5, respectively. The home bias, v, is set to match Colombia's trade openness. The consumer's utility function is assumed to be given by $u(c) = ln(c)$, and the firms' productivity follows an AR(1) process,

$$\ln z_{i,t} = \mu + \rho \ln z_{i,t-1} + \varepsilon_{i,t}$$

where $\varepsilon_{i,t}$ is normally distributed, with s.d. σ_z. Both ρ and σ_z are set such that the data generated by the model generates similar AR(1) estimates as those obtained from Colombian domestic sales data.

Regarding parameters affecting exports, the parameters τ and A^{min} are set to match the average export intensity of all exporters and the new exporters' one. f^e is set to match the share of exporters over the total active firms. The parameters α, ϕ, δ are set to match the exporters' export intensity evolution over their life cycle.

[28] I explain the algorithm to solve the model in Appendix C

Table 3: Calibrated Parameters

Parameters	Variable markups + Dynamics	CES + Dynamics	Variable markups + Static	CES + Static	Rationale
β	0.96	0.96	0.96	0.96	Yearly frequency discount rate
γ	2.5	2.5	2.5	2.5	Armington elasticity
Parameters estimated within model					
θ	2.90	3.80	2.90	3.80	”Average” price elasticity
η	4.20	-	5.60	-	Super elasticity
σ^{ω}	0.48	0.48	0.48	0.48	Firms' labor productivity s.d.
ρ^{ω}	0.61	0.61	0.61	0.61	Firms' labor productivity persistence
ν	0.71	-	0.71	-	Home bias
f_e	0.08	0.04	1.50	0.10	Exporter fixed costs
α	0.70	0.74	0.00	0.00	Customer capital: curvature
ϕ	3.72	14.30	0.00	0.00	Investment adjustment cost
δ	0.24	0.42	1.00	1.00	Customer capital: depreciation
A^{min}	0.01	0.02	1.00	1.00	Customer capital: Initial value
τ	0.44	0.20	0.38	0.61	Iceberg cost

Finally, the parameters governing the price elasticity, θ, and η, are set to lie within the markup range estimated for Colombia, and the empirical results are presented in Table 1. I perform the same exercise with the model-generated data as with the observed data, but with two exceptions. First, because I directly observe the markups in the model, I run the exact estimates as in the data, but using the markups as the dependent variable.[29] Second, following the literature, I use wage reductions as the change in the marginal cost of production. The international choke price, p^{c*}, is assumed to be a parameter consistent with the foreign demand and the estimated parameters for the price elasticity.[30] The target moments and the model predictions are presented in Table 4.

7.2 Model implications

Now, exploiting the model's ability to nest different models, I simulate and calibrate four models: with or without exporter dynamics and with or without variable markups. Then, I test each model's ability to explain the empirical facts documented in the data section.

[29]This prevents me from using the fixed effects used to control for the marginal cost changes, as explained in the empirical section.

[30]In this case, p^{c*} is assumed to be the choke price that solves the foreign economy given the foreign demand function. For this, I assume the foreign economy has the same firm distribution and price elasticity parameters, θ and η, as the domestic economy.

Table 4: Target Moments

Moment	Data	Variable markups + Dynamics	CES + Dynamics	Variable markups + Static	CES + Static
Average markup	0.45	0.44	0.35	0.45	0.35
Markup sensitivity estimates	0.65	0.63	-	0.66	-
Share of exporters	0.19	0.19	0.20	0.23	0.20
Trade openness	0.37	0.37	-	0.37	-
Av export intensity new exporters	0.40	0.40	0.16	-	-
Av. export intensity	0.45	0.50	0.23	0.46	0.25
S.d. domestic sales shocks	0.36	0.30	0.58	0.30	0.58
Persistence domestic sale shocks	0.47	0.49	0.57	0.46	0.58
Cum. growth 2nd year	0.11	0.18	0.18	-	-
Cum. growth 3rd year	0.28	0.30	0.31	-	-
Cum. growth 4th year	0.38	0.39	0.40	-	-
Cum. growth 5th year	0.39	0.45	0.47	-	-
Cum. growth 6th year	0.51	0.50	0.52	-	-

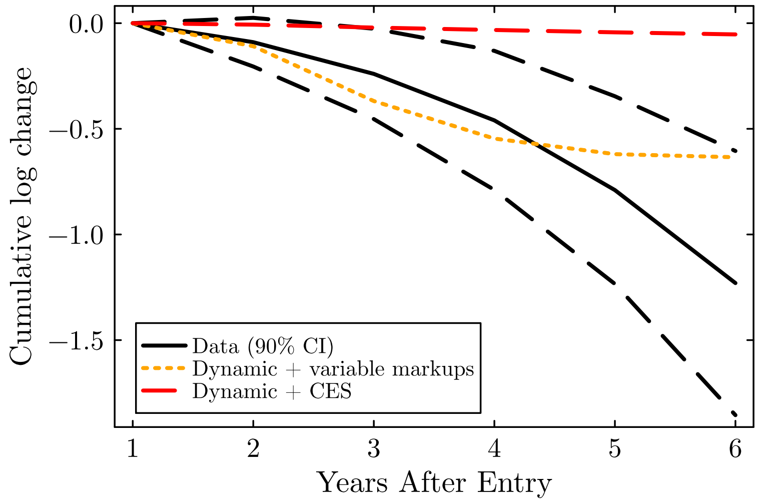
Note: Firms' cumulative growth shows the evolution of new exporters' export intensity over their life cycle; its values correspond to the estimated results shown in column 7 of Table A.5 in the appendix. Average export intensity is calculated using weighted firm-level exports. The standard deviation of domestic sales shocks and their persistence shows the standard deviation of the estimated residual and the estimated coefficient from an AR (1) estimate for firm-level real domestic sales.

The simulations only differ in the volatility of firm tfp shocks. I change the firm-level volatility parameter, σ_z, solving for the new policy functions and general equilibrium for each parameter value.[31] Note that changes in the parameter, σ_z, represent shifts in the volatility of firm tfp shocks, an exogenous input to the model. This differs from firm-level sales volatility, which, like in the empirical data, is an endogenous equilibrium outcome within the model. For consistency, when comparing the models to the data, I compute each model's s.d. of changes in domestic sales as in the data.

Quantitative result 1: Higher volatility of firms' productivity shocks reduces new exporters' growth. Figure 3 presents export intensity growth differences when exporters faced higher domestic firm-level volatility for both dynamic models. The model with variable markups adequately predicts the relationship between higher domestic firm-level volatility and the differential growth of the new exporter -yellow dotted line-, relative to data estimates -solid black line -. The model with constant markups fails in generating the observed pattern.

[31] To only change the conditional variance of the domestic sales changes, I need to adjust the mean, μ, and the persistence of the shocks ρ. Without these adjustments, the shocks affect the average firm productivity.

Figure 3: Volatility and New Exporters' Growth



Note: The orange dotted line shows the cumulative export intensity response elasticity predicted by the model with new exporters' dynamics and variable markups when the average firm domestic sales volatility increases by 1%. Both estimates from the model and data are based on the export intensity cumulative change, conditional on those exporters with at least six years of tenure in the market. The model predictions correspond to changes in the firms' productivity volatility. The data results are based on the estimates for Colombian firm-level data presented in column 6 of Table A.6.

Aggregate predictions. To understand the relevance of firm-level volatility and the proposed mechanism, let's rewrite the total exports as in section two, but now without assuming that shocks to productivity are i.i.d., in this case, total exports are given by:

$$Exp_t = \bar{A} \int_{z^*(A)} \frac{A_i^{\alpha}}{\bar{A}} \hat{rev}^*(z) d\Psi(z,A)$$

where $\hat{rev}^*(z) := p^*(z)q^*(z,1)$ is the static component of exports, and $\bar{A} := \int_{z^*(A)} A_i^{\alpha} d\Psi(z,A)$ denotes the average effective demand shifter over active exporters. Using the covariance definition and the Leibniz rule, we have that the total export response to a marginal change in a generic variable x is given by,

$$\frac{\partial \ln Exp_t}{\partial x} = \underbrace{\frac{\partial \ln \bar{A}}{\partial x}}_{\text{dynamic margin}} + \frac{1}{\Theta} \ln \left(\overbrace{\partial \frac{\mathbb{E}(rev_i^*(z)|z \geq z^*)}{\partial x}}^{\text{static margin}} + \partial \overbrace{\frac{\mathbb{C}\mathbf{ov}\left(\frac{A_i^{\alpha}}{\bar{A}}; rev_i^*(z)|z \geq z^*\right)}{\partial x}}^{\text{misallocation margin}} \right. \\ \left. \underbrace{- \frac{1}{\Theta} \int_A \frac{\partial z^*(A)}{\partial x} \frac{A_i^{\alpha}}{\bar{A}} rev^*(z^*) \psi_z(z^*, A) d\Psi_A(A)}_{\text{extensive margin}} \right)$$

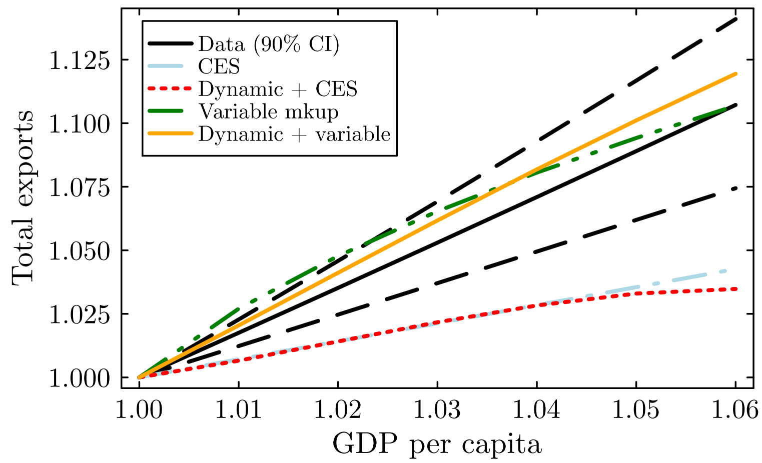
where $\Theta := \mathbb{E}(rev_i^*(z)|z \geq z^*) + \mathbb{C}\mathbf{ov}\left(\frac{A_i^\alpha}{A}; rev_i^*(z)|z \geq z^*\right)$, $\psi_z(z,A)$ denotes the conditional probability density function of firms productivity, given their value of customer capital, and $\Psi_A(A)$ is the marginal density function of customer capital.

The previous expression shows that total export reaction to changes in σ_Z takes place through the typical intensive and extensive margins. But unlike the case of static model, or dynamic version with i.i.d. productivity shocks, three sub-margins determine the intensive margin: (i) the dynamic margin capturing the changes in average customer capital ; (ii) the static margin, capturing changes in firms' export static decision- equal to the static models' total intensive margin-; and (iii) changes in the misallocation margin - absent in dynamic models with i.i.d productivity shocks-. The latter sub-margin captures changes in the covariance between firms' revenues per customer and their relative level of customer capital. A higher covariance increases exports, as it indicates that firms with higher revenues per customer are reaching relatively more customers.

Quantitative result 2: Changes in the volatility of firms' productivity shocks generates a positive relationship between exports and GDP per capita. Figure 4 illustrates the four models' predicted total trade and GDP per capita relationship, arising from changes in firm tfp shock volatility. All models predict a positive relationship between total exports and GDP per capita as we vary the firm-level volatility, consistent wit aggregate finding 1. While the four models' qualitative predictions are similar, the ones with variable markups predict a quantitative relationship closer to the conditional correlations observed in the data. The forthcoming quantitative predictions will show that the constant markup model matches this relationship, due to its counterfactual predictions between volatility, GDP per capita, trade.

Quantitative result 3: Higher volatility of firms' productivity shocks reduces total exports. Figure 5 shows the models' quantitative prediction regarding total exports and firms' sales volatility when we change σ_z as previously described. Both models with variable markups are qualitatively consistent with the documented empirical relationships between firm-level volatility and export changes driven by export cost differences. The model without variable markups predicts the opposite relationship. Within the models with variable markups, the model with exporters' dynamics generates a quantitative relationship similar to the one observed in the data. It predicts an elasticity between the domestic firm-level volatility and total exports of around 1.09,

Figure 4: GDP per capita and total exports



Note: The model predictions correspond to changes in the firms' productivity volatility. The data results are based on Table 2 where export changes are derived from the export costs relationship and translated to total exports via the model's trade elasticity.

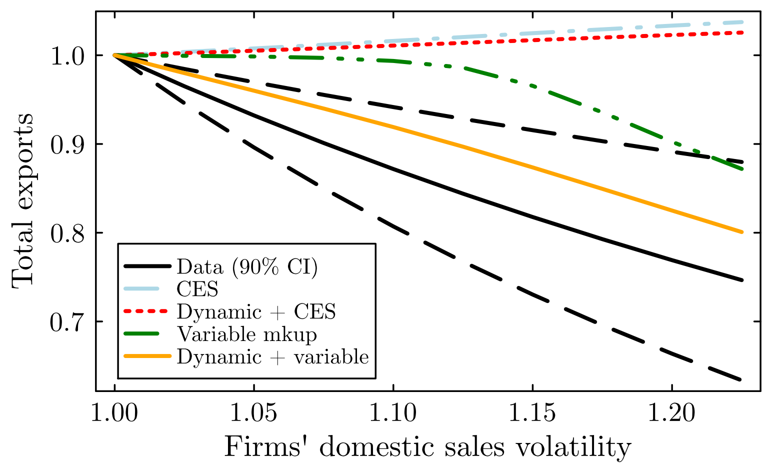
which represents 77% of the point estimates found in the empirical section - column (2) of Table 2, and is 60% higher than the model without the new exporter's dynamics.

To contextualize this finding, a reduction in firm tfp shock volatility that moves a country from the median to the first quartile of the distribution would result in an 84% increase in exports. Similarly, taking the Colombian case, if Colombian firms were to face the firm-level volatility levels of Spain or Denmark, total exports would grow by 33% and 99%, respectively. This is consistent with the puzzling observation from standard international trade models (e.g. Waugh, 2010): when abstracted from volatility changes, these models estimate higher cross-country export costs for developing economies, consistent with their higher firm-level volatility.

These results resolve the puzzling volatility-exports relationship documented by the literature (Alessandria et al., 2015; Alessandria et al., 2021; Baley et al., 2020), since it shows that abstracting from the existence of variable markups comes at the cost of missing the negative relationship between firm-level volatility and total trade. While abstracting from the existence of exporters' dynamics comes at the cost of quantitatively biasing down the negative relationship between firm-level volatility and international trade.

Quantitative result 4: Higher volatility of firms' productivity shocks reduces GDP per capita. Figure 6 shows the relationship generated in each model between firms' domestic volatility and GDP per capita, induced by changes in the volatility of firms' productivity. Rel-

Figure 5: Firm-level Volatility and Exports

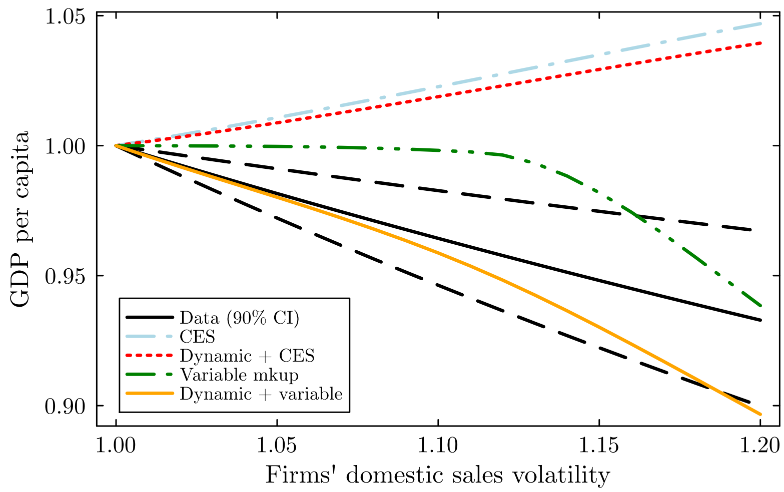


Note: Firms' domestic sales volatility refers to the standard deviation of firms' changes in domestic sales both in the model and in the data. The results are driven by underlying changes in firms productivity volatility. The data results are based on Table 2 where export changes are derived from the export costs relationship and translated to total exports via the model's trade elasticity.

ative to the relationship observed in the data, the two model versions with variable markups outperform those assuming constant elasticity. The former are quantitatively consistent with the conditional moments observed in the data, unlike the latter. This result highlights the relevance of the proposed mechanism, as it is not only quantitatively important for explaining observed trade patterns but also has significant implications for development. Specifically, the benchmark model with variable markups and new exporters dynamics predicts an elasticity of around -0.5 between firm domestic sales volatility and GDP per capita, due to changes in the volatility of firms' productivity. This implies, for example, that a reduction in the volatility of firms' productivity, which moves a country from the first or second quartile of firm-level sales volatility to the third quartile, generates a GDP per capita reduction of about 35% and 25%, respectively. These represent around 55% of the observed GDP differences between these groups of countries.

On the other hand, Figure 6 shows that both models with constant markups generate the well-known "Oi-Hartman-Abel", and hence predict GDP increases with increases in the volatility of firms' tfp shocks. This result, together with the previous one, explains why changes in the volatility of firms' productivity generate the correct relationship between GDP per capita and total exports in models with constant markup. It is due to their counterfactual predictions regarding the relationship between GDP per capita and total exports with respect to firm-level domestic volatility.

Figure 6: Volatility and GDP per capita



Note: Firms' domestic sales volatility refers to the standard deviation of the residual firms' level idiosyncratic changes in domestic sales both in the model and in the data. The model predictions correspond to changes in the firms' productivity volatility. The data results are based on Table 2

8 Conclusion

This paper shows that the firms' marginal cost volatility is a significant barrier to international trade and development. This finding arises from a general equilibrium model that incorporates two key empirically validated microeconomic features: variable demand elasticity and dynamic export investment decisions. By doing so, my findings provide a new explanation for two puzzling features of the data: the observed negative relationship between exports and firms' sales volatility and developing economies' relatively high export costs. I also find that cross-country differences in the volatility of firms' productivity are important to account for cross-country income differences.

I show how the proposed mechanism reverses the "Oi-Hartman-Abel" effect present in standard frameworks with firm heterogeneity. Higher volatility discourages exporter investment in foreign markets, hindering exporters' growth and ultimately reducing total exports and income. The model replicates the negative correlation between firm-level sales volatility and trade/income across countries, explaining a substantial portion of previously unexplained variation in export costs across development levels.

These findings demonstrate that policies promoting domestic macroeconomic and microeconomic stability can be of first-order importance in fostering international trade and development. Furthermore, the model's outcomes and its tractability open promising avenues for future research. Compared to traditional frameworks, it highlights a more prominent role for foreign trade policy uncertainty in suppressing trade and welfare. Additionally, this framework can be used to investigate the observed differences in firm distribution and growth across various stages of development.

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A Appendix

Proof of Lemma 1.

The result follows from Jensen's inequality.

Proof of Proposition 1.

Exporters' growth is given by $ln(\frac{A'}{A})$. The proposition follows from (3) and Jensen's inequality.

Proof of Proposition 2.

Let Exp^{e1} and Exp^{e2} be the total exports of two identical economies, where exporters' revenues are concave in productivity. Still, in the latter one, firms' productivity distribution is a mean-preserving spread over the firms' productivity distribution of the other.

Lemma 1 implies $Exp^{e1} > Exp^{e2}$. Denote export differences as $\Delta Exp = \ln \frac{Exp^{e1}}{Exp^{e2}} > 0$. Denote the export change predicted by the convex model as $\Delta Exp^{\text{convex model}} < 0$.

Define $\ln \hat{\tau}$ as follows,

$$\ln \hat{\tau} := \Delta Exp - \Delta Exp^{\text{convex model}} > 0$$

This implies that we need to reduce the predicted exports by the convex model by $\hat{\tau} > 1$ after a firm's productivity means preserving spread.

Denote by mgc the marginal cost such that

$$\int rev(mgc) dF = Exp^{\text{e2}}$$

Now, assume for simplicity that $Exp^{e1} = Exp^{convex,e1}$. Since revenues are continuous, define $\hat{mgc}_i = \alpha \; mgc_i$ as the marginal cost of production in the convex model, such that:

$$\int rev^{convex}(\hat{mgc}) dF = Exp^{e2}$$

It is sufficient to show that $\hat{mgc}_i > mgc_i$ for all firms. To prove it, assume the contrary. We have two cases. The first case is $\hat{mgc} = mgc$ for all firms. Since $rev(.) > 0$, $\hat{mgc} = mgc \; \forall \; i$, then $\hat{\tau} = 1$ contradicting Lemma 1.

The second case is $\hat{mgc} < mgc \; \forall \; i$. Since revenues are decreasing in mgc, we have

$$\int revrev^{convex}(mgc) dF < \int rev^{convex}(\hat{mgc}) dF$$

By definition of $\hat{mgc}$ this implies $Exp^{convex,e2} < Exp^{e2}$ a contradiction.

Hence $\hat{mgc} > mgc$, we can define the firm-level iceberg costs as

$$\tau := \alpha = \frac{\hat{mgc}}{mgc} > 1$$

Proof of Proposition 3.

The revenue change relative to the firm's productivity is as follows $\frac{drev(z)}{dz} = \frac{dp}{dz}\left[q + p\frac{\partial q}{\partial p}\right]$. The second difference is given by

$$\frac{d^2rev(z)}{dz^2} = \frac{d^2p}{dz^2}\left[q + p\frac{\partial q(p)}{\partial p}\right] + \left(\frac{dp}{dz}\right)^2\left[2\frac{\partial q(p)}{\partial p} + p\frac{\partial q(p)}{\partial p} + p\frac{\partial^2 q(p)}{\partial p^2}\right]$$

where $\frac{d^2p}{dz^2} \geq 0$, $\frac{\partial q}{\partial p} < 0$. Let $\theta < -1$ denote the price elasticity, and the elasticity of the negative of the price-elasticity to the firm's price as $\eta_{-\theta,p} > 0$. The second derivative of quantities with respect to prices is equal to:

$$\frac{d^2rev(z)}{dz^2} = \underbrace{\frac{d^2p}{dz^2}\left[q(1+\theta)\right]}_{<0} + \left(\frac{dp}{dz}\right)^2\left[2\frac{\theta q}{p} + \left[\eta_{-\theta,p} - 1 + \theta\right]p\frac{\theta q}{p^2}\right]$$

$$\frac{d^2rev(z)}{dz^2} = \underbrace{\frac{d^2p}{dz^2}\left[q(1+\theta)\right]}_{<0} + \underbrace{\left(\frac{dp}{dz}\right)^2\frac{\theta q}{p}}_{<0}\left[\eta_{-\theta,p} - (|\theta| - 1)\right]$$

When $\eta_{-\theta,p} = 0$ $\frac{d^2rev(z)}{dz^2} > 0$. But, If $\eta_{-\theta,p} > -\theta - 1 > 0 \forall z$ $\frac{d^2rev(z)}{dz^2} < 0$.

Online Appendix (For online publication only)

B Cross-country Data and Estimation Robustness

Here, I present more details about the data used for the cross-country analyses and the robustness of the cross-country estimates.

B.1 Cross-Country Estimation

Measurement of firm-level volatility. Table A.1 presents the results of estimating equation (10) using different ways of computing firms volatility. Column 1 presents the result using the baseline measure, where I only focus on firms within the manufacturing sector; column 2 presents the estimates using all firms in the sample regardless of their main sector; column 3 shows the results when restricting the sample to firms that declared zero direct or indirect exports. Column 4 presents the results when firms' volatility is constructed as in the baseline, but using the change in the number of total workers instead of sales. Table A.2 presents similar results, but using the inter-quartile range as a measure of volatility instead of using the s.d. .

One step PPML. I test for the relevance of the methodology used. Another way of estimating it is using the PPML method in one step, including a border dummy as described below:

$$\begin{aligned} sales_{ij,t} = exp\Bigg\{ & I_{\text{Border}_{i\neq j,t}} + I_{\text{Border}_{i\neq j,t}} \times \Bigg(\beta_1 \ln \sigma_{i,t} + \beta_2 \ln \frac{GDP_{i,t}}{L_{i,t}} + \beta_3 \ln \text{fin. fric}_{i,t} \\ & + \boldsymbol{\beta_4} \boldsymbol{y}_{ij,t} + \boldsymbol{\beta_5} h_{i,t} \Big) + \beta_6 \ln \sigma_{i,t} + \beta_7 \ln \frac{GDP_{i,t}}{L_{i,t}} + \beta_8 \ln \text{fin. fric}_{i,t} \\ & + \boldsymbol{\beta_9} \boldsymbol{y}_{ij,t} + \boldsymbol{\beta}_{10} h_{i,t} + \gamma_{j,t} + \varepsilon_{ij,t} \Big\}; \end{aligned} \tag{25}$$

where $sales_{ij,t}$ includes both domestic and bilateral sales. The main variable of interest is the one described in the main text when interacted with a border effect dummy $I_{\text{Border}_{i\neq j,t}}$. The dummy equals one when i sales to a foreign country, and zero when sales are domestic, allowing me to identify the differential effect of our variable of interest in trade relative to domestic flows. The vector $\boldsymbol{y}_{ijt}$ includes standard gravity controls for each bilateral country pair: (1) log distance between most populated cities (km); (2) UN diplomatic disagreement score; and indicator variables for (3) contiguous borders; (4) common official or primary language; (5) for when at least 9% common language; (6) for past colonial ties; and (7) for when a free trade agreement is in place. Lastly, the vector $h_{i,t}$, denotes a control for firms' entry costs - proxy by the numbers of procedures to register a business- and a set of indicator capturing if the country belongs to the European union, the world trade organization, is a GATT membership, and two categorical variables for the origins and the current the legal system type. Finally, $\gamma_i t$ denotes destination-year fixed effects. Table A.3 presents the estimation results. Previous findings remain valid.

B.2 Firm-level Estimation

Estimation of Markup response: Instrumental variable approach

In this appendix section, I present more details of the estimation procedure to estimate the markups' response to exchange rate changes and additional robustness checks.

As already mentioned in the main text, to test whether markup changes vary with exporters' market share, we can estimate

$$\Delta p_{i,d,l,t} = \beta_1 \Delta e_{i,d,l,t} \times \text{exp. share}_{i,d,l,t-1} + \beta \text{exp. share}_{i,d,l,t-1} \times \mathbf{X} + \gamma^1_{i,l,t} + \gamma^2_{i,l,d} + \gamma^3_{l,d,t} + e_{i,d,l,t} \tag{26}$$

To estimate β without bias, we need to abstract from the exchange rate variation that might reflect changes in the average productivity of the destination country, as this can bias the estimate. I use an instrumental variable approach that solves this concern. I instrument the bilateral exchange rate variation intersected with the firm's sales shares with the remittances flows from third countries to Colombia, interacted with firms' sales shares to that destination. The first stage is then given by

$$\begin{aligned}\Delta e_{i,d,l,t-1} \times \text{exp. share}_{i,d,l,t-1} = \Delta remittances_{d,t} \times \text{exp. share}_{i,d,l,07} + \\ + \beta \text{exp. share}_{i,d,l,t-1} \times \mathbf{X} + \gamma^1_{i,l,t} + \gamma^2_{i,l,d} + \gamma^3_{l,d,t} + e_{i,d,l,t}\end{aligned}$$

Two assumptions are needed to validate this procedure. First, remittance flows to Colombia need to affect the exchange rate of Colombia with the rest of the countries; this seems natural as the average net remittances to Colombia represent, on average, 10% of the total export flow. Also, it has been documented that the remittances are unlikely to vary due to exchange rate variation as seems mainly driven by income variation.[32] The second assumption is that shocks affecting the remittances to Colombia from a third country do not generate differential price changes for a product sold in several destinations after controlling for the common shocks that may hit all

[32]See Mandelman (2013) for a discussion on the effect and relevance of remittances on the exchange rate.

products in the destination countries. We need that, conditional on the destination market shocks and the firm's common marginal cost at the product level, the changes in remittance flows from a third country cannot be related to shocks affecting the relative differences in firms' prices to different destination markets.

Results comparing the IV and the OLS estimation are presented in Table A.7. Columns 1 and 3 show the OLS results, while the rest of the column presents the results using the IV strategies. As can be seen, the F-statistic ranges between 65.15 and 108.22 for different specifications. Relative to Table 1 presented in the main text, the current one adds four additional results in Columns 5 to 7. Column 5 presents the results of dropping the firm-destination-product fixed effects. Column 6 shows that the results hold if we condition the sample on those exporters that continue exporting in the following period. Column 7 presents the findings after re-estimating the IV strategy, fixing the exporter's share in 2012, and using data between 2013 and 2019 to re-do the estimates. The three columns show that results are invariant to these changes and that the benchmark case (column 4) used to calibrate the model is on the conservative side of the estimates.

New exporters' dynamics

To estimate what drives exporters to grow into foreign markets, I estimate:

$$\Delta_h y_{i,d,l,t} = \sum_{h=0}^{6} \beta_h \mathbb{I}^h_{\{age=h\}} + \ln p_{i,d,l,t} + \gamma^a_{i,l,t} + \gamma^b_{d,l,t} + \gamma^c_{\text{cohorts}} + \varepsilon_{i,d,l,t} \ ,$$

where $\Delta_h y$ represents the log differences between the initial value of the variable y and its value "h" years after, I estimate the above equation for two possible dependent variables: one $\text{export}^q_{i,d,l,t}$ representing the total export quantities that firm i is selling of product l to destination d in year t; the other is $\frac{\text{export}_{i,d,l,t}}{\text{Tot. sales}_{i,t}}$, representing nominal exports from firm i to each market at time t over total sales.[33] I project variable y against a dummy variable $\mathbb{I}^h_{\{age=h\}}$ that equals one when the exporters spent h years continuously selling product l to destination d. I control for the prices of the product, $p_{i,d,l,t}$, firm-product-time fixed effects, $\gamma^a_{i,l,t}$, and product-destination-time fixed effects $\gamma^b_{d,t}$. Adding these fixed effects allows me to purge out the common variation in sales from firm i of product l at time t to all markets; the second set of fixed effects allows me to purge out the common variation across exporters within a destination product time. In my benchmark

[33]Ideally, I would want to divide by the total domestic product sales for the same product l, but that data is unavailable

specification, $\gamma^{c}_{\text{cohorts}}$ represents the first month of entry I observed. To understand the price dynamics over the exporter's life cycle, I estimate the same equation but without controlling for prices:

$$\ln p_{i,d,l,t} = \sum_{h=0}^{6} \beta_h^p \mathbb{I}^h_{\{age=h\}} + \gamma^a_{i,l,t} + \gamma^b_{d,l,t} + \gamma^c_{\text{cohorts}} + \varepsilon^p_{i,d,l,t}$$

In this case, β_h^p captures the differential changes in prices over the life cycle of the exporter relative to the common variation in prices for that product l at time t.

By construction β_1^p and β_1 are set to zero so that each estimate of $\{\beta_h\}_{h=1}^H$ or $\{\beta_h^p\}_{h=1}^H$ captures the cumulative change of the dependent variable to the exporter entry value. New exporters are those exporters that did not export any positive amount to that product-destination market in the last three years.[34]

Results are presented in Table A.5. Columns 1 to 4 show the estimation results using changes in quantities exported, columns 5 to 8 present the cumulative changes in export intensity, and columns 9 to 10 show the cumulative changes in prices over the exporters' life cycle. Results show that exporters tend to increase their exports and export intensity slowly, conditional on prices, while they do not seem to adjust relative prices across destinations. Consequently, exporters grow by shifting the intercept of their demand.

Volatility and exporter life cycle

Baseline firm's sales volatility measure. I proceed as before, but now I exploit a leave-one-out strategy, taking advantage of a more detailed Colombian firm-level database. As before, firms' i idiosyncratic sales changes are estimated as follows:

$$\Delta\text{Domestic sales}_{i,j(i),t} = \gamma_{j(i),t} + \Delta\hat{s}^D_{i,j(i),t}$$

where $\Delta\text{Domestic sales}_{i,t}$ denotes the log difference of domestic sales over a year, $\gamma_{j(i),t}$ denotes industry-time fixed effects - $j(i)$ is firm's i industry-, and $\Delta\hat{s}^D_{i,t}$ firms' i idiosyncratic sales changes.

I compute the firms' exposure to domestic firm-level volatility, $\sigma_{i,t}$, as follows. I compute the average cross-sectional of firm-level shocks $\Delta\hat{s}^D_{i,t}$, at time t, for all the firms in the same industry except for i. The focus on domestic sales shocks to third firms within the same industry obeys

[34] This implies that I lost the first three years of my sample since I cannot observe if the exporters did any export before.

two reasons: first, it allows me to avoid the volatility measure being related to the direct effects of foreign demand shocks, and second, it prevents the measure from being related to shocks to the firm itself.

Below, I detail how other firms' exposure measures to volatility are constructed:

Robustness measure 2: A product weighted measure of firms' volatility exposure. The measure of volatility used in Column (8) is constructed as follows:

1. Compute the log difference on one year of the real domestic sales of each firm i, defined as Δdom. sales

2. Compute the cross-section standard deviation of Δdom. sales, for each year for those firms with the same main export products at the sixth digit belonging to the product category J. And take the average over time for each 6-digit product j. Denote this measure by sd_J^{hs6}

3. Compute the 6-digit product export share over the corresponding 4-digit products for each firm i.

4. Compute the 6-digit product average share as the average share between 2006 and 2009 for all the firms selling that 6-digit product.

5. Use the 6-digit product average computed in the previous step to weight the volatility computed in step 2 for each firm-4-digit product.

Robustness measure 3: Firm-level common shocks to construct volatility measure. The measure of volatility used in Column (9) is constructed as follows:

1. Restrict the sample to those exporters with at least two products and two countries of destination.

2. First, compute the common changes in the exports of a firm i in time t, $\gamma_{i,t}$, to all its products and destinations it sells to by estimating:

$$\Delta exp_{i,l,d,t} = \gamma_{i,t} + \theta_{d,l,t} + e_{i,l,d,t} \tag{27}$$

3. Compute the cross-section standard deviation of $\gamma_{i,t}$, for each year t, of those firms other than i with main export products in the 6-digit category that belong to the product cat-

egory J. Take the average over time for each 6-digit product j. Denote this measure by $sd_J^{hs6,Common}$

Robustness measure 4: Product-specific shocks to construct volatility measure. The measure of volatility used in Column (10) is constructed as follows:

1. Restrict the sample to those exporters with at least two products and two countries of destination.
2. First compute the export firm-destination-product shocks, $\Delta e\hat{x}p_{i,l,d,t}$ by estimating:
$$\Delta exp_{i,l,d,t} = \theta_{d,l,t} + \Delta e\hat{x}p_{i,l,d,t} \tag{28}$$
3. Compute the cross-section standard deviation of $\Delta e\hat{x}p_{i,l,d,t}$ for all the firms other than i selling that 6-digit product.
4. Use the volatility in the previous step to take the firm-level average volatility.

Estimation results. Table A.6 presents the estimation of equation (8). Columns 1 to 7 present the results using the baseline measure for domestic sales volatility. Columns 5 to 7 estimate equation (8) conditional on those exporters with at least five and six years of tenure. Column 7 presents the results using the cumulative changes in total exported quantities instead of export intensity. Columns 8 and 10 use different measures of firm-level volatility explained above.

The similarity of the documented patterns suggests that the results are not driven by the possible selection due to firms' exit, nor by the measure of volatility used, nor by the dependent variable used to measure customer capital evolution. When I test for all these cases, the documented patterns are similar, and I can't reject the model's main predictions regarding the effects of volatility on exporters' life cycle evolution.

C Model Algorithm

The model only needs to solve for the economy's steady state given different parameters for σ and its counterpart adjustment in μ and ρ, such that we only do a mean-preserving spread over the conditional volatility of firms' productivity.

Given the high non-linearities of the firm's problem, I solve the model using global methods. First, the firms' domestic decisions are static, and we only need to solve them for optimal prices. To solve the export decision, firms need to know their customer capital level A, their productivity z_i, and domestic wages, w, with which they need to make a proper forecast for z_i', and w'. In principle, firms need to know the firm's distribution to solve for w and w'. However, because I solve for the steady distribution, rather than using the firms' distribution as a state variable, which is infeasible, I use wage prices as a state variable, which is sufficient to characterize the firm's decision, given the assumption of a small open economy.

To solve for the economy's aggregate equilibrium, I proceed as follows: When calibrating the model, I set the wage equal to one. This allows me to set wages equal to one in the baseline economy without any changes. For each change in the volatility parameters, I re-solve for the entire value function, policy functions, and aggregate economy.

For each parameter value, the solution is computed as follows:

1. Fix the parameter values of the problem. and pre-set ε to small value.

2. Set a grid space of $(20X85X10)$ for firms' productivity, customer capital, and wages. Solve for the optimal value function and optimal policy function using global methods.

3. Pre-set wages to w^n

4. Use the obtained optimal policy function to expand the grid space to $(100X120)$ possible grid points for the state variable. Compute a Markov transition matrix for the firms' measure for state variable, $H(.)$, conditional on wages w^n

5. Pre-set a non-degenerate aggregate distribution Ψ^j, conditional on wage w^n

6. Update Ψ using the Markov transition matrix until $|\Psi^{j+1} - \Psi^j| \leq \varepsilon$

7. Using Ψ, compute the aggregate variable and the domestic choke price p_d^c

8. Compute the excess labor demand $\Delta L = L^d - L^s$.

9. If the labor excess demand $|\Delta L| > \varepsilon$, update $w^n = w^{n+1}$ and start from 3 again.

I set a wage level, and using the expanded space, I compute the Markov transition matrix for each firm state based on the firms' optimal decisions, conditional on the preset wage. Using the transition matrix, I can update the aggregate distribution until it converges, given a wage. Then, after solving for all the equilibrium objects, I can construct labor demand and supply curves and check if the labor market is in equilibrium. If not, I adjust the wages and start the process again.

Appendix Tables

Table A.1: Firm-level Volatility, Development and Exports Costs

	(1)	(2)	(3)	(4)
ln (GDP per capita)	-0.447***	-0.547***	-0.592***	-0.561***
	(0.114)	(0.124)	(0.126)	(0.122)
ln (Firm volatility)^{Baseline}	0.577***			
	(0.101)			
ln (Financial frictions)	0.086	0.077	0.082	0.074
	(0.126)	(0.143)	(0.148)	(0.142)
ln (Firm volatility)$^{\text{All firms}}$		0.499***		
		(0.113)		
ln (Firm volatility)$^{\text{No exporters}}$			0.414***	
			(0.111)	
ln (Firm volatility)$^{\text{workers}}$				0.516***
				(0.105)
N	126	126	126	126
adj. R^2	0.590	0.547	0.526	0.548
Year FE	✓	✓	✓	✓
Controls	✓	✓	✓	

Note: Table reports estimates of equation (10) with varying firm-level volatility measures: manufacturing sample (baseline), all firms sample, non-exporters sample (direct or indirect exports), and worker-based volatility. Export costs are estimated using PPML, following a gravity specification similar to Waugh (2010) (equation (9)). Annual trade flows are used. Controls include financial access (as indicated in the WBES database), entry costs (measured by the number of procedures), and dummies for EU, WTO, GATT membership, and legal origin. Standard errors clustered at the origin country level are in parentheses. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table A.2: Firm-level Volatility (IQR), Development and Exports Costs

	(1)	(2)	(3)	(4)	(5)
ln (GDP per capita)	-0.684***	-0.383***	-0.392***	-0.447***	-0.424***
	(0.135)	(0.114)	(0.111)	(0.111)	(0.118)
ln (Financial frictions)	0.070	0.065	0.089	0.084	0.068
	(0.159)	(0.122)	(0.123)	(0.129)	(0.127)
ln (Firm volatility)$^{IQR-manuf}$		0.401***			
		(0.060)			
ln (Firm volatility)$^{IQR-all}$			0.419***		
			(0.077)		
ln (Firm volatility)$^{IQR-no\ exp}$				0.388***	
				(0.079)	
ln (Firm volatility)$^{IQR-workers}$					0.460***
					(0.079)
N	126	126	126	126	126
Year FE	✓	✓	✓	✓	✓
Controls	✓	✓	✓	✓	✓

Note: Table reports estimates of equation (10) with varying firm-level volatility measures: manufacturing sample (baseline), all firms sample, non-exporters sample (direct or indirect exports), and worker-based volatility. Volatility measures are constructed using the inter-quartile range (IQR). Export costs are estimated using PPML, following a gravity specification similar to Waugh (2010) (equation (9)). Annual trade flows are used. Controls include financial access (as indicated in the WBES database), entry costs (measured by the number of procedures), and dummies for EU, WTO, GATT membership, and legal origin. Standard errors clustered at the origin country level are in parentheses. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table A.3: One step PPML

	(1)	(2)	(3)	(4)
ln (GDP per capita)	0.259*	0.491***	0.514***	-0.057
	(0.134)	(0.129)	(0.134)	(0.124)
$I_{\text{Border}} \times \ln(\text{GDP per capita})$	0.591***	0.522***	0.539***	0.636***
	(0.146)	(0.144)	(0.148)	(0.133)
$\ln(\text{Firm volatility})^{Baseline}$	-0.783***			
	(0.115)			
$I_{\text{Border}} \times \ln(\text{Firm volatility})^{Baseline}$	-0.255**			
	(0.120)			
$\ln(\text{Firm volatility})^{All}$		-0.650***		
		(0.110)		
$I_{\text{Border}} \times \ln(\text{Firm volatility})^{All}$		-0.234**		
		(0.113)		
$\ln(\text{Firm volatility})^{no\ exp}$			-0.513***	
			(0.100)	
$I_{\text{Border}} \times \ln(\text{Firm volatility})^{no\ exp}$			-0.311***	
			(0.102)	
$\ln(\text{Firm volatility})^{IQR\ manuf.}$				-0.717***
				(0.073)
$I_{\text{Border}} \times \ln(\text{Firm volatility})^{IQR\ manuf.}$				-0.276***
				(0.077)
Observations	26516	26516	26516	27250
Year × Destination FE	✓	✓	✓	✓
Controls × I_{Border}	✓	✓	✓	✓

Note: Table presents PPML estimates of equation (25) using annual domestic and trade flows. It presents results using different volatility measures as detailed in Appendix B.1. Gravity controls for each pair include log distance, UN disagreement score, and dummies for contiguous borders, common official language, at least 9% common language, past colonial ties, and free trade agreement. Origin country controls include financial access (WBES), entry costs (procedures), GDP per capita (PPP), and dummies for EU, WTO, GATT, and legal origin. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table A.4: Robustness Firm-level Volatility and Exports

	Dependent variable: Av. Exp							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(GDP per capita)	1.81***	1.27***	0.92***	1.23***	0.98***	0.92***	0.94***	0.93***
	[0.20]	[0.15]	[0.16]	[0.16]	[0.15]	[0.17]	[0.16]	[0.16]
ln(Micro Volatility)			-1.44**		-1.62***			
			[0.62]		[0.49]			
ln(Micro Volatility$^{tfp}_{NonExpo}$)						-0.96**		
						[0.46]		
ln(Micro Volatility$^{\text{tfp}}_{All}$)							-0.84*	
							[0.46]	
ln(Micro Volatility$^{\text{Common}}$)								-0.14**
								[0.07]
Observations	35211	35211	35211	35211	35211	35211	35211	35211
R^2	0.75	0.85	0.91	0.89	0.93	0.93	0.92	0.93
Number of countries	38	38	38	38	38	38	38	38
Gravity Controls	Size	All	All	All	All	All	All	All
Doing Business	-	Exp	Exp	All	All	All	All	All

Note: The table replicates the results of Table 2 using different ways of computing firm-level volatility. Standard errors are in brackets and are clustered at the origin country level. $^{*}p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.5: Exporters Life Cycle

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\exp^q$	$\exp^q$	$\exp^q$	$\exp^q$	$\frac{exp}{\text{Tot. sales}}$	$\frac{exp}{\text{Tot. sales}}$	$\frac{exp}{\text{Tot. sales}}$	$\frac{exp}{\text{Tot. sales}}$	$\ln(p)$	$\ln(p)$
$\mathbb{I}_{\{age_{ildt}=2\}}$	0.11***	0.13***	0.16***	0.21***	0.06***	0.08***	0.11***	0.17***	-0.02	-0.02
	[0.02]	[0.02]	[0.02]	[0.04]	[0.02]	[0.02]	[0.02]	[0.04]	[0.01]	[0.02]
$\mathbb{I}_{\{age_{ildt}=3\}}$	0.25***	0.31***	0.39***	0.59***	0.17***	0.20***	0.28***	0.42***	0.01	0.02
	[0.03]	[0.03]	[0.04]	[0.06]	[0.03]	[0.03]	[0.04]	[0.08]	[0.02]	[0.02]
$\mathbb{I}_{\{age_{ildt}=4\}}$	0.35***	0.43***	0.55***	0.68***	0.22***	0.26***	0.38***	0.48***	0.00	0.04
	[0.03]	[0.04]	[0.05]	[0.09]	[0.03]	[0.04]	[0.05]	[0.09]	[0.02]	[0.03]
$\mathbb{I}_{\{age_{ildt}=5\}}$	0.37***	0.48***	0.59***	0.60***	0.22***	0.29***	0.39***	0.34**	0.02	0.01
	[0.05]	[0.06]	[0.07]	[0.14]	[0.04]	[0.05]	[0.07]	[0.15]	[0.03]	[0.04]
$\mathbb{I}_{\{age_{ildt}=6\}}$	0.43***	0.50***	0.72***	0.65***	0.29***	0.32***	0.51***	0.46**	-0.00	-0.01
	[0.07]	[0.08]	[0.09]	[0.15]	[0.06]	[0.08]	[0.10]	[0.18]	[0.04]	[0.05]
$\mathbb{I}_{\{age_{ildt}=7\}}$	0.45***	0.49***	0.73***	0.90***	0.39***	0.43***	0.63***	1.17***	-0.01	-0.03
	[0.13]	[0.14]	[0.17]	[0.32]	[0.14]	[0.14]	[0.18]	[0.41]	[0.06]	[0.09]
Observations	55,315	51,950	37,061	17,254	52,446	49,129	34,650	51,950	17,254	15,381
R^2	0.18	0.31	0.41	0.58	0.15	0.25	0.36	0.88	0.97	0.53
Year × Dest. FE	✓	✓	✓	-	✓	✓	✓	-	✓	-
Year × Product FE	✓	✓	-	-	✓	✓	-	-	-	-
Year × Firm FE	-	✓	-	-	-	✓	-	✓	-	-
Year × Firm × Product FE	-	-	✓	✓	-	-	✓	✓	✓	✓
Year × Product × Dest. FE	-	-	-	✓	-	-	-	✓	-	✓

Note: New exporters entered the export market and have not exported that 6-digit product to that destination in at least the past three years. All exporters are continuing exporters each year. Error cluster at the destination country. $exp.^q$ denotes the use of quantities cumulative change as dependent variable (columns 1 to 4), $\frac{exp}{\text{Tot. Sales}}$ use the ratio of nominal exports to domestic sales instead (columns 5 to 8). Columns 9 and 10 use prices as the dependent variable. Error cluster at the 6-digit product. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.6: Volatility and Exporters Life Cycle

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	Δexp^q	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$	$\Delta\frac{\text{exp}}{\text{Tot. sales}}$
$\mathbb{I}_{\{age_{ildt}=2\}} \times \ln \text{Vol.}$	0.01	-0.01	0.00	0.00	-0.01	-0.09	-0.05	0.01	-0.09*	0.06
	[0.03]	[0.03]	[0.04]	[0.04]	[0.06]	[0.07]	[0.06]	[0.02]	[0.05]	[0.09]
$\mathbb{I}_{\{age_{ildt}=3\}} \times \ln \text{Vol.}$	0.00	0.00	-0.00	-0.02	-0.11	-0.24*	-0.19**	-0.01	-0.07	-0.03
	[0.04]	[0.04]	[0.05]	[0.06]	[0.09]	[0.13]	[0.09]	[0.03]	[0.07]	[0.13]
$\mathbb{I}_{\{age_{ildt}=4\}} \times \ln \text{Vol.}$	-0.08	-0.08	-0.10	-0.09	-0.25*	-0.46**	-0.27**	-0.06	-0.11	-0.14
	[0.05]	[0.05]	[0.07]	[0.09]	[0.13]	[0.20]	[0.12]	[0.04]	[0.10]	[0.17]
$\mathbb{I}_{\{age_{ildt}=5\}} \times \ln \text{Vol.}$	-0.21***	-0.21***	-0.26***	-0.34***	-0.56***	-0.79***	-0.63***	-0.18***	-0.35**	-0.79***
	[0.08]	[0.08]	[0.10]	[0.12]	[0.18]	[0.27]	[0.20]	[0.06]	[0.17]	[0.26]
$\mathbb{I}_{\{age_{ildt}=6\}} \times \ln \text{Vol.}$	-0.41***	-0.46***	-0.54***	-0.66***	-0.82***	-1.23***	-0.74***	-0.21***	-0.88***	-0.88**
	[0.14]	[0.14]	[0.17]	[0.19]	[0.29]	[0.38]	[0.28]	[0.08]	[0.30]	[0.42]
$\mathbb{I}_{\{age_{ildt}=7\}} \times \ln \text{Vol.}$	-	-	-	-	-	-	-0.86**	-	-	-
	-	-	-	-	-	-	[0.35]	-	-	-
Observations	24,038	23,930	23,349	17,496	11,364	8,326	13,141	17,513	17,502	18,789
Year × Dest. FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Year × Product FE	-	✓	✓	✓	✓	✓	✓	✓	✓	✓
Year × Firm FE	-	-	Only Firm	✓	✓	✓	✓	✓	✓	✓
Year × Firm × Product FE	-	-	-	✓	✓	✓	✓	✓	✓	✓
Total tenure	≥ 3	≥ 3	≥ 3	≥ 3	≥ 5	≥ 6	≥ 5	≥ 3	≥ 3	≥ 3
Measure	Bench.	Bench.	Bench.	Bench.	Bench.	Bench.	Bench.	Measure 2	Measure 3	Measure 4

Note: The table presents the estimation of equation (8). Columns 1 to 7 use the benchmark measures of domestic exposure to volatility. Column 7 uses the change in export quantities, denoted by $\Delta exp.^q$, instead of the changes in export intensity. Columns 8 to 10 use the other measures of volatility described in B.2. Total tenure refers to the minimum number of years exporters have continuously exported to each market in the sample. Error cluster at the firm level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.7: Robustness for Markups Estimates

	(OLS)	(IV)	(OLS)	(IV)	(IV)	(IV)	(IV)
Panel 1: First Stage							
	Dependent variable: Δex. rate$_{d,t}$ $\times$ $share_{i,l,d,t-1}$						
	-	$\Delta e \times share$	-	$\Delta e \times share$	$\Delta e \times share$	$\Delta e \times share$	$\Delta e \times share$
Δremit$_{\cdot \neq d,t}$ $\times$ $share_{\cdot,07}$		0.28***		0.28***	0.29***	0.28***	0.36***
		[0.03]		[0.03]	[0.03]	[0.03]	[0.04]
Panel 2: Second Stage (Prices)							
	Dependent variable: $\Delta \log p$						
Δexchange rate $\times$ *share*	0.11	0.82***	0.09	0.65**	0.59**	0.69**	1.15**
	[0.08]	[0.29]	[0.10]	[0.29]	[0.28]	[0.30]	[0.56]
Panel 3: Second Stage (Quantities)							
	Dependent variable: $\Delta \log q$						
Δexchange rate $\times$ *share*	0.77***	-3.21***	0.25	-2.09***	-3.21***	-2.17***	-3.34***
	[0.21]	[0.70]	[0.21]	[0.62]	[0.70]	[0.62]	[0.80]
Observations	62,357	62,357	58,781	58,781	58,781	57,774	45,053
F-statistic	-	80.68	-	101.81	108.22	97.16	65.15
Firm-product-time FE	✓	✓	✓	✓	✓	✓	✓
Destination-product-time FE	✓	✓	✓	✓	✓	✓	✓
Firm-product-Destination FE	-	-	✓	✓	-	✓	✓
Controls $\times$ $share_{i,l,d,t}$	Agg. prices	Agg. prices	All	All	All	All	All
Continue exporting in $t+1$	-	-	-	-	-	✓	-
After year 2012	-	-	-	-	-	-	✓

Note: Columns 1 - 4 are the same as Table 1. Continued exporting denotes the case when the sample is restricted to exporters that continue to export in the following year. The year 2012 denotes the robustness case when export shares are fixed in 2012, and the sample is taken after 2012. Standard errors are in brackets. Error cluster at the destination country. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$